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Physics. — *Determination by X-rays of density and axial ratio of hexagonal silver iodide.* By N. H. KOLKMEIJER, W. J. D. VAN DOBBENBURGH and H. A. BOEKENOOGEN. (Communicated by Prof. ERNST COHEN.)

(Communicated at the meeting of May 26, 1928).

I. *Introduction.*

According to the "Strukturbericht 1913—1926" of EWALD and HERMANN [1]) three modifications are known of silver iodide, which can be distinguished by *X*-rays. They are: a hexagonal and a cubical, both stable below 146°, and a cubic one stable above 146°. One of us three [2]) made a physicochemical investigation of silver iodide. Very accurate determinations of density were made of different preparates, which gave as maximum 5.68[3]. A greater value was not found, so that it was reasonable to take for granted that the preparate with this density was physically pure.

One of us [3]) has pointed out that by means of *X*-ray exposures it is generally possible to judge if a preparate is physically pure. The density namely can be deduced from the photos in question. If then the same value is found as was found by another method, f.i. pycnometrically, we can in most cases conclude with certainty that the preparate is physically pure.

In order to know therefore if the silver iodide, mentioned above, with a density of 5.68[3] was physically pure, it seemed desirable to determine the density also with the help of *X*-rays.

II. *Data from the literature.*

These are in extenso to be found in the thesis of one of us, which thesis is already cited above. Suffice it here to say that according to GROTH [4]) a hexagonal form (Iodyrite) is found in nature. By the help of *X*-rays WILSEY [5]) and DAVEY [6]) found cubic structure in a synthetic preparate below 146°, although indications of a hexagonal form were

[1]) P. P. EWALD and C. HERMANN. Strukturbericht 1913—1926. Z. Krist. 1927.

[2]) W. J. D. VAN DOBBENBURGH, Thesis, Utrecht 1928. ERNST COHEN and W. J. D. VAN DOBBENBURGH, Z. f. phys. Chem. **137**, 289, 1928.

[3]) N. H. KOLKMEIJER, Proc. Acad. Amst. **31**, 151 (1927). Z. f. phys. Chem. **136**, 45, 1928.

[4]) P. GROTH, Chemische Kristallographie I, 200 (1906).

[5]) R. B. WILSEY, Phil. Mag. **42**, 262 (1921); Phil. Mag. **46**, 487 (1923).

[6]) W. P. DAVEY, Phys. Rev. **19**, 248 (1922).

present. Only the latter was found by AMINOFF[1]). He also found that above 146° a cubic form is stable, though it is not identical with the cubic form found by WILSEY and DAVEY. Between the two forms, which are stable below 146°, it is difficult to distinguish by means of *X*-rays with the DEBYE-SCHERRER method, because most of the lines of one modification coincide with those of the other.

III. *Our own powder-photos.*

In the first place we made a powderphoto of a preparate of silver-iodide which we supposed to be physically and chemically pure α—*AgJ* (density: 5.68[3]) on account of the values found for the density.

We also made a photo of a preparate the density of which was 5.49[5], because we hoped to find here at any rate a great percentage of the other modification.

The photos were made with CuK-rays. The β-radiation was filtered out with a nickelfoil, attached to the end of the lead diaphragm (with a bore of 1 mm.) projecting outside the camera. The preparate was in a glass tube, thickness of glass 0.01 mm. This tube was always centered in the camera. We purposely avoided powdering our preparates. Powdering can considerably alter the density of certain preparates of silver iodide, as had appeared already, while it is also known that powdering transforms zinc-blende into WURTZITE[2]). (With these two types of structure in fact we are concerned in the case of *AgJ*). By omitting powdering we ran the risk that no continuous lines, but only series of dots would be found in the photos. We avoided this hazard by regularly rotating the glass tube with the preparate. Usually an exposure of 10 hours was found to be sufficient, when a current of about 12 m.A. passed through the tube. The thickness of the preparate was 1.5 mm. The radius of the film in the camera was 2.75 cm.

The same glass tube was used for the photos of the two preparates; after the first exposure it had been washed with a strong KCN-solution, in order to remove crystallization nuclei of the old preparate. The two photos obtained in this way are reproduced in fig. 1. Number I is the one with the iodide $d_{4°}^{30°} = 5.68^3$, Number II of the iodide with $d_{4°}^{30°} = 5.49$.

Under these two photos (fig. 2) is a reproduction of which the upper half is taken from photo I, the lower half from photo II. The lines which signify most are indicated as *a*, *b*, *c*, *d*, *e*, and *f*. As is seen photo II shows another line *b* between the lines *a* and *c*, and another *e* between *d* and *f* which neither appear in photo I[3]).

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[1]) G. AMINOFF, Geologische Föreningens i Stockholm Förhandlingar **44**, 444 (1922).

[2]) A. SCHLEEDE und H. GANTZCKOW, Z. f. Phys. **15**, 184 (1923).

[3]) Pretty well the same thing may be found very clearly reproduced in Zeitschrift für Physik **15**, 187 (1923) for the corresponding case of zincblende and wurtzite.

These lines also appeared in an exposure of silver iodide with a density of $d_{40}^{30°} = 5.62$, with less intensity however.

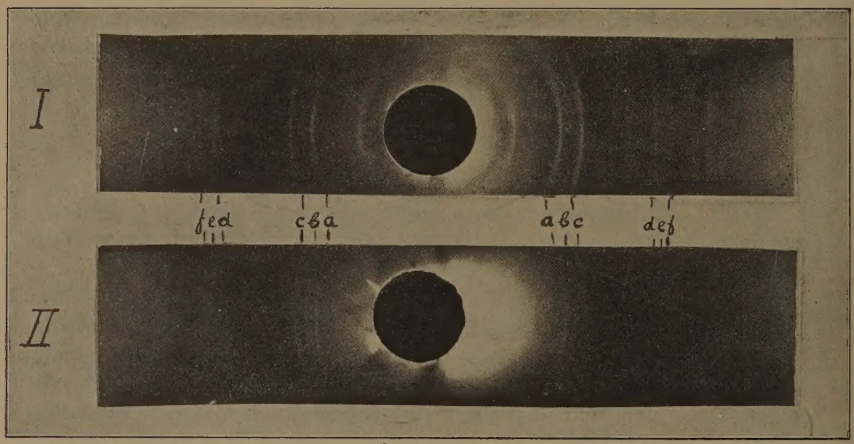

Fig. 1.

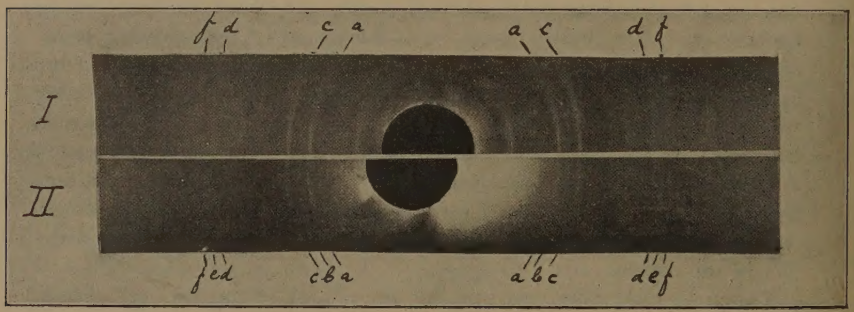

Fig. 2.

In. fig. 3 we find, placed one over the other:

1. the places of the lines, calculated from the hexagonal structure with the lattice parameters, obtained from the measurements which are described further on;

2. the places of the lines, as WILSEY gives them for the hexagonal structure however reduced to CuKα radiation;

3. the places of the lines according to our photo II;

4. the places of the lines according to our photo I;

5. the places of the lines as WILSEY gives them of the cubic structure however reduced to CuKα radiation;

6. the places of the lines calculated from the cubic structure with the

lattice parameters, which are obtained when the cubic structure is produced from the hexagonal structure by shifting in their own planes the lattice planes, perpendicular to the ternary axis.

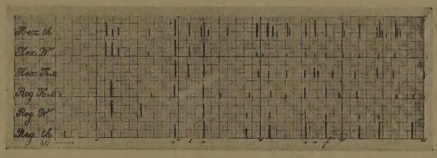

Fig. 3.

The figure has been drawn to a scale agreeing to the full size of our photos. We indicate the intensities by the length of the lines, in five arbitrary classes.

As may be seen from fig. 3 the cubic and the hexagonal structures have many lines in common. Yet the impression is given that the structure of iodide with $d_{45}^{30°} = 5.68^3$ agrees best with the cubic structure, though a few lines of the hexagonal modification seem to be very faintly present, and, what is most remarkable, not the strongest [1]). The photo of preparate VIII with $d_{45}^{30°} = 5.49$, on the contrary, shows two lines which decidedly belong to the hexagonal structure, namely lines *b* and *e*. There is no doubt that these two lines are present in the original photo II, while we did not succeed in finding them in photo I. Now it will be clear from fig. 3 that several lines, especially with the hexagonal structure, are very close together, so that in the photo they are confused, and show themselves as rather broad and badly defined lines. Therefore we will not bring forward a definite statement about the structure of our preparates, but only indicate what we think most probable on the evidence of our powder photos.

1. We presume that the iodide which we [2]) described thus far as *α-AgJ* possesses the cubic regular structure.

2. The iodide with lower density also contains *AgJ* with the hexagonal structure.

We found it impossible to ascertain, from the readings of the photos, a value for the density with satisfactory accuracy. Dr. A. E. VAN ARKEL at Eindhoven also experienced difficulties in the solution of this problem. We thought it therefore desirable to make some exposures with the help of the spectrographic method.

IV. *Photos of the* 001 *plane with the spectrographic method.*

A natural hexagonal crystal out of the collection of the Mineralogical-

[1]) We must allow for the possibility that by the influence of Röntgenradiation the transformation cubic hexagonal may take place.

[2]) W. J. D. VAN DOBBENBURGH, Thesis, Utrecht 1928.

Geological Institute at Utrecht was put at our disposal. We wish to thank Prof. RUTTEN once more for the loan of this fine specimen. It showed the 001 plane to the magnitude of nearly 1 cm^2. The other planes however were very imperfectly developed, and we did not succeed in getting good reflections on these other planes. Finally the firm of C. GOLDBACH at Zell a.H. supplied us with a specimen with an exceedingly good 101-plane. The photo of the 101 plane was made with this crystal. For the photo of the 001 plane the crystal first mentioned was mounted below a crystal of gypsum on the table of a goniometer, in such a way that the reflecting planes of the two crystals were coplanar, and contained the axis of rotation of the goniometer. One arm wore a slit 0.2 mm. wide, made of lead of a thickness of 3 mm. The slit was at a distance of 28 cm. from the axis of rotation. On the same arm, but closer to the axis of rotation, stood a diaphragm, likewise made of lead of a thickness of three mm., which served to screen off the undesired radiation emanating from the slit. A second arm carried a camera, a brass box, the face of which was covered with black paper, and in which the film was mounted light-tight. The table on which the crystals were mounted was rocked [1]) over an angle of about 3 degrees during the exposure. We also used here *CuK*-rays, this time however without filtration of the β-rays. The reflection of the 001-plane was taken in the 2nd and the 6th order. The 101-plane was taken in the 1st order only. Of the two photos first mentioned, reproductions are given in fig. 4. The plates were measured with the help of a ZEISS', so called "small Abbe-comparator"

The $CuK\alpha_1$- and the α_2-line in plate 74 are marked 1; the $CuK\beta$-line is marked 2, all relating to *AgJ*; the corresponding lines for gypsum are marked 3 and 4. All these lines are of the second order.

In plate 75 the $CuK\alpha_1$- and α_2 line of the sixth order for *AgJ* are marked 1; 3 marks the corresponding lines of the sixth order for gypsum; 4 marks the β-line of the seventh order. We wish to observe that, in the reproduction, line 4, which is present in the original, is not to be found. We indicate the place of it approximately with a small dash. With the *AgJ* we chose the second and the sixth order, because these two, according to the calculation for the hexagonal structure, were expected to be pretty strong, whereas the other orders were either absent or had too small an intensity.

A difficulty presented itself in the measurement because the lines of the iodide were not between the reference lines of the gypsum, but above them. At first we hoped that the sliding-movement of the comparator used in the direction of the lines would be sufficiently accurate. This proved however not to be the case, so that we were compelled

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[1]) It is well known that by so doing irregular crystals also give very fine lines on the plate by focussing. See i.a. DE BROGLIE. C. R. **157**, 924, 1413 (1913); **158**, 177 (1914).

to draw in some way or other a straight line on the plate, as much as possible parallel to the lines we wished to measure. All the readings could then be referred to this straight line, so that one is independent

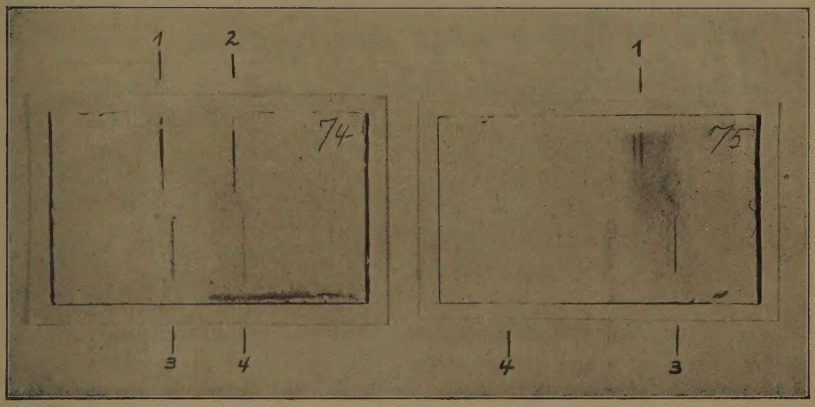

Fig. 4.

of the sliding-movement of the comparator. After trying a few other methods, but unsuccessfully, we found that the following method was usable: namely by cementing with two drops of piceïne a thin thread of glass to the plate, as much as possible parallel to the lines.

When measuring the lines it was seen that after a little practice an adjustment was possible with an accuracy of about 5 micron with the distinct lines, while greater divergencies were found with the lines that were less well defined. We shall now give the results of the measurements on plate 74, while, as an example, we shall give the calculation of this plate in extenso. We always made 14 measurements at the top of a line, and also 14 at the bottom of it. The averages deduced from it differed with two observers almost constantly 8 μ. In this manner the distances of the lines were known up to 2 μ accurate. We indicated the place of these measurements on the plate with a very slight inkdot, so that the distance between these dots in the direction of the lines could also be determined. In measuring we first of all determined the place of

	Thread	α_2	α_1	β	Thread	Place of the reading
Ag J	30.427	40.863	41.179	52.437[5]	30.426	24.8
	30.453	40.882	41.131	52.403	30.450	30.0
gypsum	30.619	42.357	42.600	53.717	30.616	39.5
	30.613	42.260	42.530	53.703	30.622	44.1

the thread of glass, then made the measurements for the lines α and β, and then checked whether the place, originally found for the thread of glass, was found back.

Distance of the thread to the middle of the plate 17.5 mm. The distances of the different lines to the thread are:

Place of the reading	α_2	α_1	β		Thread
24.8	10.436[5]	10.752[5]	22.011	Ag J	0
30.0	10.370[5]	10.679[5]	21.951[5]		0
39.5	11.739[5]	11.982[5]	23.099[5]	gypsum	0
44.1	11.642[5]	11.912[5]	23.085[5]		0

We can calculate the inclination of the thread with respect to the lines from the place of the reading and the differences between each line and the thread at the top and at the bottom. From our observations we find for the angle which the thread makes with the lines: 0.0129 radians. So we can calculate, assuming that the top of the thread remains unchanged, how much we must change the distance between the lines and the thread, if we think this thread turned parallel to the lines.

For these corrected distances of the lines to the thread we find:

Place of the reading	α_2	α_1	β		Thread
24.8	— 10.436[5]	— 10.752[5]	— 22.011	Ag J	0
30.0	— 10.437[5]	— 10.746[5]	— 22.018[5]		0
39.5	— 11.929[5]	— 12.172[5]	— 23.289[5]	gypsum	0
44.1	— 11.891[5]	— 12.161[5]	— 23.334[5]		0
Mean	— 10.437	— 10.749[5]	— 22.015		
	— 11.910[5]	— 12.167	— 23.312		

This gives for the distance to the middle of the plate, the above-mentioned distance 17.5 being allowed for:

α_2	α_1	β	
7.063	6.750[5]	— 4.515	Ag J
5.589[5]	5.333	— 5.812	gypsum

For the calculation we did not use the wavelengths as they are given in SIEGBAHN [1]), because (SIEGBANN l. c., p. 28): „All the wavelengths given in the following tables have been calculated with the aid of the uncorrected BRAGG law". We made the correction in the following way:

The corrected BRAGG law [2]), namely, runs: $2d \sin {}^1/_2 \vartheta \left(1 - \frac{b}{n^2}\right) = n\lambda$.

For d, the grating constant of rocksalt, we take the value which has been taken as basis by all spectrocopists, $d = 5.628$ Å, and for b the value for this substance which is given by COMPTON [3]), namely: $d = 9.6 \times 10^{-5}$. The wavelengths (λ_s) given by SIEGBAHN are found according to the formula $2d \sin {}^1/_2 \vartheta = n\lambda_s$.

It follows that $\lambda = \lambda_s \left(1 - \frac{b}{n^2}\right)$. So we find:

	λ_s	λ
$Cu\ K\alpha_2$	1541 . 16	1541 . 12
$Cu\ K\alpha_1$	1537 . 30	1537 . 26
$Cu\ K\beta$	1389 . 33	1389 . 30

For gypsum we take the grating constant $d = 7.578$ Å [4]), [5]). So we must find the angles of deflection ϑ from $2d \sin {}^1/_2 \vartheta \left(1 - \frac{b_{gypsum}}{n^2}\right) = n\lambda$.

Now it is necessary to know the value for b_{gypsum}. COMPTON [6]) states that for crystals as rocksalt and calcite (at least for $\lambda < 1.5$ Å, which is about what we have here) the b may be calculated from the following formula:

$$b = \frac{2\, se^2 d^2}{\pi mc^2},$$

in which is:

s = number of electrons per cubic cm,
e = charge of electron,
d = grating constant,
m = mass of the electron,
c = velocity of light.

This is allowed because the natural frequencies of the electron move-

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[1]) M. SIEGBAHN, The spectroscopy of X-rays, London 1925, p. 105.

[2]) In the following table the values deduced differ somewhat from those given in the thesis written by one of us, in consequence of the correction in BRAGG's law being applied now.

[3]) A. H. COMPTON, X-rays and Electrons, New-York, 1926, p. 212

[4]) H. MARK, Die Verwendung der Röntgenstrahlen in Chemie und Technik. Leipzig 1926, p. 447.

[5]) The coefficient of expansion of gypsum, perpendicular on the cleavage plane is so very slight (1.13×10^{-6} Tables of ABRAHAM and SACERDOTE, Paris 1913) that we need not allow for the difference in temperature.

[6]) A. H. COMPTON, l. c., p. 212.

ments for all the atoms present in those crystals are much smaller than those of the incident rays.

This must however also hold good for gypsum $CaSO_4 . 2H_2O$. So we find for $b_{gypsum} = 7{,}4 . 10^{-4}$. That this calculation is not much in error is seen from the fact that the same calculation for calcite gives $b_{CaCO_3} = 1.35 . 10^{-4}$ while COMPTON [1]) gives for it $1.46 . 10^{-4}$.

So we find for the second order of gypsum:

$$\begin{aligned} {}^1/_2\,\vartheta\,\alpha_2 &= 11^\circ\ 44'.2 \\ {}^1/_2\,\vartheta\,\alpha_1 &= 11^\circ\ 42'.4 \\ {}^1/_2\,\vartheta\,\beta &= 10^\circ\ 34'.0 \end{aligned}$$

The distance measured on the plate between the α_1- and the β-line of gypsum therefore corresponds with an angle of $2 \times (11^\circ 44'.2 - 10^\circ 34'.0) = 2^\circ 16'.8$. If we call the distance of the photographic plate to the axis of the goniometer r; if the α_1- respectively β-radiation, reflected on the crystal of gypsum, forms with the normal on the photographic plate (line which connects the axis of the goniometer with the middle of the plate) angles φ_1 and φ_2, then:

$$tg\,\varphi_1 = \frac{5.333}{r} \quad \text{en} \quad tg\,\varphi_2 = \frac{-5.812}{r}$$

while

$$\varphi_1 - \varphi_2 = 23^\circ\ 24'.6 - 21^\circ\ 7'.8 = 2^\circ\ 16'.8.$$

It follows that:

$$tg\ 2^\circ 16'.8 = \frac{\frac{5.333}{r} + \frac{5.812}{r}}{1 - \frac{5.333}{r} \times \frac{5.812}{r}}$$

From this quadratic equation we calculate: $r = 280.03$ mm. By means of this value we find $tg\,\varphi_3 = \frac{7.063}{r}$ etc. (for all the 6 lines) and from this the angles φ_3 etc. themselves. These angles are:

$1^\circ\ 26'.7$	$1^\circ\ 22'.9$	$-0^\circ\ 55'.4$
$1^\circ\ 8'.6$	$1^\circ\ 5'.5$	$-1^\circ\ 11'.4$

As the α_1-ray, reflected on the gypsum, deviates an angle of $1^\circ 5'.5$ from the direction towards the middle of the plate, while its deviation from the transmitted radiation amounts to $2 \times 11^\circ 42'.4 = 23^\circ 24'.8$, we know now that the direction towards the middle of the plate deviates $22^\circ 19'.3$ from the direction of the transmitted ray. This allows us to find the real angles of deviation ϑ for the different radiations. They are:

$23^\circ\ 27'.9$	$23^\circ\ 24'.8$	$21^\circ\ 7'.9$ (*Ag J*)
$23^\circ\ 46'.0$	$23^\circ\ 42'.2$	$21^\circ\ 23'.9$ (*gypsum*)

[1]) A. H. COMPTON, l. c., p. 212.

This last line serves as a check. It ought to run:

$$23^\circ\ 28'.4 \qquad 23^\circ\ 24'.8 \qquad 21^\circ\ 8'.0$$

The deviation of the α_2-line finds its explanation in the fact that the middle of this weaker line has perhaps been wrongly estimated on account of the immediate neighbourhood of the strong α_1-line. That this deviation however is of little moment is seen from the fact that by means of this value $23^\circ 27'.9$ we calculate for the lattice constant of gypsum: 7.579, whereas we started from 7.578.

From the values found for ϑ we find by applying BRAGG's law:

$$2\,d\,.\,sin\ {}^1/_2\,\theta = n\lambda \text{ with } n = 2$$

$$d = 7.484 \quad 7.485 \text{ and } 7.483, \qquad \text{mean: } 7.484 \text{ Å}$$

which value must be corrected for the deviation of BRAGG's law.

Such a calculation applied to plate 75 gives:

$$d = 7.489^8 \times 10^{-8} \text{ cm. and } 7.489^0 \times 10^{-8} \text{ cm.}, \qquad \text{mean } 7.489 \text{ Å}.$$

From the two values found for $c = 7.484$ from the second order, and $c = 7.489$ from the sixth order, we can now deduce a value for the correction coefficient b. We have namely:

$$c\left(1 - \frac{b}{4}\right) = 7.84 \quad \text{en} \quad c\left(1 - \frac{b}{36}\right) = 7.489.$$

From this we calculate $b_{AgJ} = 0.0030$, and as definite value for $c = \mathbf{7.490}$ Å. The value for b seems to be rather large. Yet it is of the right order of magnitude. We have, namely, also tried to calculate b from the theoretical formula, for which however we could not use the form used above, as AgJ does not comply with the conditions mentioned for this calculation. Here we must therefore use the complete formula [1]), which for our purpose we easily convert into:

$$b = \frac{2\,e^2 d^2}{\pi m c^2} \sum_s \frac{n_s}{1 - (\nu_s/\nu)^2}$$

Here n_s signifies the number of electrons of a certain kind per cubic cm.; ν_s their own frequency, ν the frequency of the incident radiation, while the summation refers to the different kinds of electrons.

COMPTON [2]) advises to choose for ν_s the frequency of the sharp end of the absorptionband for every kind of electrons in the atom. We cannot expect, however, that the result is certainly the correct one. Making this choice we find the following table, in which we have taken for ν $\frac{c}{1.539 \times 10^{-8}}$. (See Table next page).

The result is b_{AgJ} (theoretically) $= 0.0015$.

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[1]) A. H. COMPTON, l. c., p. 209.

[2]) A. H. COMPTON, l. c., p. 210.

	n_s [1]	λ_s [1]	$(\nu_s/\nu)^2$	$\frac{n_s}{1-(\nu_s/\nu)^2}$
Ag	2 × 2	0.4850	10.069	− 0.44106
	2 × 2	3.2605	0.2228	+ 5.1467[5]
	2 × 2	3.5047	0.1928	4.9554
	2 × 4	3.6844	0.1745	9.6910
	2 × 2	17.1	0.0081	4.0326
	2 × 2	21.0	0.0054	4.0217
	2 × 4	22.1	0.0048[5]	8.0390
	2 × 4	32.7	0.0022	8.0177
	2 × 6	33.2	0.0021[5]	12.0258
	2 × 2	120	0.0002	4.0008
	2 × 2	280	0	4.0000
	2 × 4		0	8
	2 × 4		0	8
	2 × 6		0	12
	2 × 1		0	2
J	2 × 2	0.3737	16.960	− 0.25063
	2 × 2	2.3819	0.4175	+ 6.8669
	2 × 2	2.5483	0.3647	6.2963
	2 × 4	2.7124	0.3219	11.798
	2 × 7	(11.2)	0.0189	4.0771
	2 × 2	13.21	0.0136	4.0552
	2 × 4	14.07	0.0120	8.0972
	2 × 4	19.48	0.0062	8.0498
	2 × 6	19.82	0.0060	12.072
	2 × 2	(62)	0.0006	4.0024
	2 × 2	(84)	0.0003	4.0012
	2 × 4	(84)	0.0003	8.0024
	2 × 4	230	0	8
	2 × 6	240	0	12
	2 × 2		0	4
	2 × 2		0	4
	2 × 3		0	6
			$\Sigma \frac{n_s}{1-(\nu_s/\nu)^2} = 204.56$	

[1]) For n_s and λ_s are chosen the values, given in COMPTON, l.c. For a few missing values we have interpolated between the values given for elements with atomic numbers one higher and one lower.

As we could not have expected a better agreement, the difference with the value found above $b_{AgJ} = 0.0030$ does not seem to be too excessive. So we shall continue to work with the number which we have determined by experiment:

$$b_{AgJ} = 0.0030$$

V. *Exposure of the 101-plane according to the spetrographic method.*

We took the photo of the 101-plane in the same way as of the 001-plane, but a crystal of calcite was substituted for the crystal of gypsum.

For the calculation we used as grating constant of calcite the value given by COMPTON [1]) for 18° (mean temperature during the exposure) $d = 6.0575$, and for $b = 1.46 \times 10^{-4}$ [2]). We then find for

$$^1/_2\,\vartheta\,\alpha_2 = 14^\circ\ 44'.4$$
$$^1/_2\,\vartheta\,\alpha_1 = 14^\circ\ 42'.1$$
$$^1/_2\,\vartheta\,\beta = 13^\circ\ 15'.6$$

Here the distance of the α_2-line to the β-line of calcite therefore represents an angle of $2 \times (14^\circ 44'.4 - 13^\circ 15'.6) = 2^\circ 57'.6$.

In the same way as described above we find for $r = 276.59^5$ for one of the two observers; for the other $r = 276.58$.

This gives for $2d$:

7.0136 and 7.0128 for one observer, and 7.0123 and 7.0115 for the other; mean 7.0125^5.

A correction to $d = 3.506^3$ must be made, as this d represents $d_{101}(1 - b)$. This b can be calculated from the b for the 001-plane found above; namely: $b_{101} = b_{001} \dfrac{d_{101}^2}{d_{001}^2}$.

This gives $b_{101} = 0.00066$. Thus we find for $d_{101} = \mathbf{3.508^6}$.

From the formula

$$\frac{1}{d^2} = \frac{4}{3a^2}(h_1^2 + h_2^2 + h_1 h_2) + \frac{1}{c^2}(h_3^2)$$

follows for the 101 plane

$$\frac{1}{(3.508_6)^2} = \frac{4}{3a^2} + \frac{1}{(7.490)^2}$$

This gives $a = \mathbf{4.585^6}$ and the axial ratio $c/a = \mathbf{1.633^4}$.

The density d_4^{18} is found from the formula

$$d_4^{18} = \frac{2 \times M.G. \times 10}{6.061 \times 10^{24}} \times \frac{1}{^1/_2\, a^2\, c \sqrt{3}}$$

and is seen to be: $d_4^{18} = \mathbf{5.681}$.

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[1]) A. H. COMPTON, l. c. p. 327.

[2]) A. H. COMPTON, l. c., p. 212.

Concerning the axial ratio 1.633[4] we wish to observe that it differs but slightly from the amount which is deduced from the wurtzite structure, if we take for granted that the tetrahedra present in it are regular, namely: 1.633°.

On the other hand ZEPHAROVICH [1]) gives for the axial ratio from the crystallographic determinations = 1.63920.

It seems to us, however, that it is not allowed to deduce from the mutually not a little different values of the measurements an exact number to so many decimals. We are of opinion that the value given by us is to be preferred.

We see that the density of 5.681 found by us for the hexagonal modification differs very little from the maximal density 5.68[3] found pycnometrically for *AgJ*, and it seems as if this good agreement supports the two determinations. Let it be borne in mind, however, that we thought we might conclude from the powderphotos that the substance with the density of 5.68 contained chiefly the cubic modification.

From what precedes we therefore cannot draw the conclusion that the preparate with the density of 5.68[3] was physically pure. From the values given by WILSEY and DAVEY for the cubic modification we can now calculate that the density of the latter would be 5.677. That this value differs so little from the value found by us for the hexagonal modification, was to be expected, for, if $c/a = 1.633$, as is the case here, the zincblende structure may be obtained from the wurtzite structure by shifting the lattice planes perpendicularly to the ternary axis, each in its own plane. Therefore, the densities of the two modifications being pretty well the same, it is impossible to conclude, on the strength of the density alone, that we have a physically and chemically pure silver iodide. Here we have the case to which attention was drawn by one of us [2]) that Röntgenphotos do not allow of drawing a conclusion in the matter of the physical purity of a substance if the densities of the two modifications differ but little. Yet it seemed to us that we might come to a conclusion by observing the preparate, with a density of 5.683 in a polarisation microscope. If namely the preparate is absolutely cubic, as we expected, the field of the polarisation microscope will remain dark. The preparate, ($d = 5.49$) containing much more hexagonal material, will have to reveal itself, partly or wholly, as optically anisotrope. And indeed the preparate ($d = 5.50$) did show distinct illuminated patches as compared with the preparate ($d = 5.68$); the difference was greater than between rocksalt and the preparate $d = 5.68$. This agrees very well with the powder-photograms, in which the preparate with a density of 5.49 did give the lines of the hexagonal form, and the preparate, with a density of 5.68 on the contrary, did not.

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[1]) VON ZEPHAROVICH, Z. Krist. **4**, 119 (1880).

[2]) N. H. KOLKMEIJER l.c.

We wish to conclude from what has been said, that the preparate with a density of 5.68[3] is approximately physically pure cubic silver iodide. and the preparate with smaller density probably physically pure hexagonal silver iodide. It seems to us that probably the lower density is to be attributed to vacuoles.

Utrecht, May 1928. VAN 'T HOFF *Laboratory*.

Physics. — *Physical Purity and Powderröntgenogram.* II. By N. H. KOLKMEIJER. (Communicated by Prof. ERNST COHEN.)

(Communicated at the meeting of September 29. 1928).

Concerning the question whether the red and the yellow mercury oxides are different modifications or not there is a difference of opinion between ERNST COHEN on one side [1]) and a few other investigators [1]) — among whom WI. OSTWALD — on the other. G. R. LEVI [1]) thinks that he has been able to decide this question; he found, namely, that the two forms of HgO give identical powderphotos with Röntgenrays, and concludes from it that they are not different modifications. I [2]) have pointed out that it is not allowed to come to this conclusion from the observation made. LEVI [3]) denies that my objections have any value, and in terms which show that he has completely misunderstood my meaning. It seems therefore necessary to try and elucidate my line of thought.

In my previous communication concerning this question reproductions are given of a powderphoto of physically pure white tin, and of a powder photo of a mixture of 90 % of white and 10 % of grey tin. The two photos are identical, and yet there are undeniably two modifications of tin.

From the investigations by ERNST COHEN and his co-workers we know now that, when preparing chemically pure substances, there is a very great chance to get mixtures of modifications, and that it is also very difficult to make a preparate that is physically pure.

At any rate there is a chance that the preparate "red HgO" consists of f.i. 99 % of a red modification and 1 % of a yellow, and that the preparate "yellow HgO" consists of 90 % of the red and 10 % of the yellow modification. And if this should be so, it is possible, as appears from the experiment with the white and grey tin, mentioned above, that the preparates of red and yellow HgO give identical powderphotos. But then we cannot conclude from this that there is only one form of HgO, just as we cannot do so with tin either. If LEVI wants his conclusions to be received, he will have to demonstrate that a similar case as the present has not arisen with his preparates.

Evidently seeing this LEVI has now used crystals of red HgO as preparate for the Röntgenexposure, and has convinced himself, by the use

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[1]) ERNST COHEN; WI. OSTWALD and others; G. R. LEVI. Cf. the citations in my paper mentioned in footnote 2).

[2]) N. H. KOLKMEIJER. Proc. Acad. Amst. **31**, 151, 1927. Z. f. phys. Chem. **136**, 45, 1928.

[3]) G. R. LEVI. Gazz. chim. Ital. **58**, 417 (1928).

of the microscope, that they were pure. We must indeed admit that a crystal is a physically pure preparate if it satisfies the following three conditions. 1st It must not be a mixed crystal; 2nd it must not contain inclusions of mother liquor with possibly other modifications; 3rd it must not be modified by incipient polymorphous transformation on the faces. Of course it is not sufficient to convince oneself with the microscope alone that the conditions, mentioned above, have been satisfied.

But even if we admit that this preparate was physically pure and then learn from LEVI that the powderphoto of it is identical with the one of another undetermined preparate "yellow HgO", we can conclude from it only, that the yellow preparate f.i. consists at least for 90 % of the red modification, but not that there cannot exist a separate yellow modification. LEVI's demonstration would be perfect only when he should get the same lines on the photo, if he made an exposure of physically pure yellow HgO.

I have other objections against much that is treated in LEVI's paper [1]). But as this is not to the purpose I refrain from discussing it. I cannot agree, however, with any of the four conclusions which LEVI mentions in his summary.

VAN 'T HOFF-*Laboratory.*

Utrecht, October 1928.

[1]) Especially where he is of opinion that I have given a wrong interpretation of M. R. ANDREWS' conclusion in Phys. Rev. **17**, 26 (1921). The author in question has ascertained that the powderphoto of a mixture of 80 % of iron and 20 % of nickel does not show the lines of nickel separately. I wish to point out that I have not said, as LEVI pretends, that the absence of the Ni-lines is the consequence of the want of sensitiveness of the method. On this point too, LEVI has misunderstood my meaning,

Embryology. — *On the Development of the Tail in the Amphibian Embryo.* By Miss J. H. BIJTEL and Prof. M. W. WOERDEMAN.

(Communicated at the meeting of September 29, 1928).

The development of the hindmost part of the body has been frequently recorded in the literature. This is not surprising, if we bear in mind that this development is closely related to the gastrulation-process, one of the problems of embryology that have given rise to most controversy.

Of late years the views regarding the gastrulation of amphibian embryos have considerably improved by the application of new methods of investigation. It is above all W. VOGT [1]) who has added many important contributions to our knowledge of the process of gastrulation. He applied pieces of agar-agar, saturated with a solution of a vital dye to the surface of blastulae and so succeeded in putting small local marks on it. Because the cellgroups, which had attracted the dye, retained it for a long time, VOGT could watch the fate of circumscript parts of the blastulae during and after the gastrulation. Coincidently with a better conception of the gastrulation the opinions concerning the formation of the hindpart of the body changed. It is outside the scope of the present paper to dwell at large upon the literature relating to this subject. But for a clear understanding of what follows we consider it well to say something here about the hitherto almost universal conception, and about the opinion of W. VOGT.

According to the present views the different organs (nervous system, notochord, mesoderm-segments) existing in the tail, are supposed to originate from an indifferent cell-mass, the tail-bud, which is supposed to comprise elements of the three germ-layers, and to develop posteriorly from the region of the blastopore, where these three germlayers are connected with each other. In this indifferent mass differentiation is believed to appear during the growth, so that nervous-system, notochord, myotomes, etc. become visible and thus this mass should add one segment after the other to the existing trunk.

The results of W. VOGT's researches, however, are different. He states this most explicitly, when he says on page 66 of his publication: "Chorda und Medullaranlage werden nach hinten in ihre Schwanz-abschnitte nicht durch Auswachsen verlängert, sondern durch Streckung, wobei das Hinterende dieser Anlagen nach Medullarschluss zunächst vom After gelöst und immer weiter nach hinten entfernt wird." Now BRACHET [2]) a.o.

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[1]) W. VOGT: Verhandl. Anatom. Gesellschaft. Ergzh. Anatom. Anz. **61**, blz. 62, 1926.

[2]) A. BRACHET: Traité d'Embryologie. Paris, MASSON & Cie. 1921.

had maintained that at the end of the gastrulation strictly speaking only the head of the Amphibian embryo is present (acro- and cephalo-genesis have terminated), and the entire trunk and tail are still to be developed (notogenesis) by apposition from an indifferent mass (zône of growth). VOGT's observations led to the conception that at the end of the gastrulation trunk and tail are already present, but that they obtain their ultimate form through displacement of material and stretching ("Gestaltungsbewegungen").

BRACHET's idea justifies the assumption that directly after the end of the gastrulation the part of the neural plate, lying right in front of the blastopore, corresponds with a place in the fully developed nervous system, which we may imagine to be just a little behind the N. vagus. According to VOGT the anlage of the whole nervous system then already exists on the neural plate, and the caudal end of the nervous system develops through stretching of that part of the plate which lies right in front of the blastopore.

As regards the formation of the mesoderm of the tail, VOGT imparts that just before the closure of the blastopore the invagination-process, characterizing the gastrulation in Amphibians, is still going on in the lateral and the ventral lip of the blastopore. The material which is invaginated then furnishes the somites of the posterior part of the trunk, growing out from the lateral lip, whereas proceeding from the ventral lip, mesoderm of the side-plate in the environment of the anus, and non-segmented mesoderm for the tail is formed. The somites of the tail, however, arise from those parts of the neural folds that encompass the blastopore. They are not used for the formation of the central nervous system. On their coalescence an ectodermsuture appears that will procure the dorsal as well as the ventral part of the caudal fin.

It appears from the above that in the formation of the tail VOGT assumes a stretching for notochord and neural tube, whereas the mesoderm is formed by adding new material to the already existing.

At the same time we may deduce from VOGT's other arguments that during the stretching of notochord and neutral tube material is supplied from an indifferent mass, which does not take part in the formation of the caudal ectoderm, but it does in that of the neural tube, notochord, somites, and non-segmented mesoderm. This mass is used up during the formation of the tail and consequently cannot be considered as a "tail-bud", or a "centre of growth".

This indifferent material is provided by the lips of the blastopore through further invagination of the cell-groups that originally lay on the surface of the lips. The final result of his research is summarized by VOGT in the following words: "Damit scheint mir erwiesen, dasz das hintere Körperende und speziell die Schwanzknospe nicht einem örtlichen Wachstumszentrum, sondern einer Gestaltungsbewegung ihre Entstehung verdankt; die Materialverschiebung, die sich dabei abspielt, ist die Fortsetzung der während der Gastrulation begonnenen Mesoderminvagination; longitu-

dinale Streckung und Konvergenz des Materiales nach der dorsalen Mitte sind die weiteren Hauptmomente, die der Gestaltung des Hinterendes zugrunde liegen und dieselbe als Fortsetzung der Gastrulationsbewegungen verstehen lassen."

By marking Amphibian embryos with vital dyes, one of us, Miss BIJTEL, has carried out an investigation into the prospective significance of the various parts of the neural plate, and has recorded it already in a short memoir [1]). The caudal part of the neural plate has also received her attention, and she was thereby induced to consider the tail-problem. Her observations give a somewhat different picture of the tail-formation from that of VOGT. Before long they will, as is to be hoped, be published in detail. It will do to report here only a few general results of her research.

Whe, therefore, take a single example out of the many experiments on embryos of Amblystoma mexicanum, Triton taeniatus, as well as Rana esculenta. Now there are no essential differences between Urodela and Anura, as the marking experiments bore out, but there is only a difference in degree. Since there is no unanimity on this subject and BRACHET [2]) has tried, after the appearance of VOGT's study, to partly account for the difference in their opinions by the different circumstances viz. that VOGT worked with Urodela and he himself with Anura, it will be well to state here at once that, although we speak about a case relating to Urodela, in principle the same can be found with experiments on Anura.

In an embryo of Amblystoma mexicanum with a neural plate, that begins to close already, and with a slitlike blastopore three stained marks are put the one behind the other in the manner shown in Fig. 1*a*. The hindmost lies before and by the side of the blastopore, and overlaps the extremities of the neural folds; the other two marks lie more in front and also overlap the folds. The middle mark being of a different colour from the other two, the development of the areas of each of the marks can very well be watched. In Fig. 1*b* we see that the neural folds are moving towards each other, and will soon coalesce. In Fig. 1*c* we see the embryo after the coalescence of the folds is accomplished. (Side view of the embryo.) As the marks also stretched over part of the folds, the ectodermal suture, originated with their fusion, is also coloured. Now, in the living animal (Fig. 1*d* and 1*e*) it could be observed that the area of the anterior mark is considerably elongated. The middle mark has stained the ectoderm of the end of the tail and a small portion of the dorsal and the ventral margin of the tail-fin, while the hindmost mark can be seen again quite on the ventral margin of the tail. Fig. 1*e* shows that in the living animal also coloured somites are looming through the ectoderm and we can see that the hindmost somites have the colour of the central mark and the segments lying between the last-named and the unstained trunk-segments, have the colour

[1]) J. H. BIJTEL: Nederl. Tijdschr. v. Geneesk. 72. I. 26. 1928.
[2]) A. BRACHET: Arch. f. Entw. mechan. 111. 1927.

of the other two marks. After the embryo had got a little older, it was prepared under an anaesthetic, i.e. after the ectoderm had been prepared away, also the myotomes were removed, in order to observe the staining of the neural tube and notochord. The results of this inquiry have been

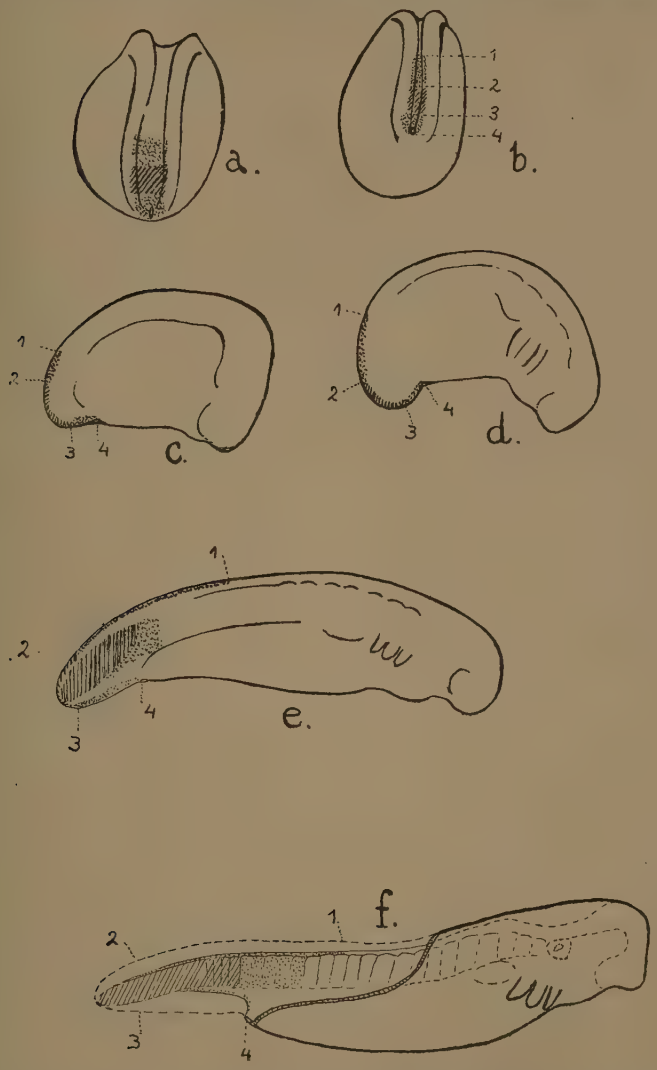

Fig. 1. Six serial developmental stages of an embryo of Amblystoma mexicanum; stage *a* provided with three stain-marks. In *f* part of the skin has been removed. (Zeiss. Object. $F = 55$. Ocul. 2).

The figures 1, 2, 3 and 4 indicate the limits of the stain-marks.

brought together in Fig. 1*f*. First of all let us point to the very large extension of the ectoderm-area, which has arisen from the coalescence of the neural folds in the region of the anterior mark. Only at the tip of the tail has the ectoderm been formed from the central mark. This area also extends a little way over the dorsal margin of the tail, while it overlaps the ventral margin only for a very small distance. Almost the whole ventral margin has originated from the ectoderm of the posterior mark. From this it may be concluded that in the stage illustrated by Fig. 1*a* part of the ectoderm of the tail already exists on the neural folds, and is elongated during its further development. There is no question about a complement of this part of the ectoderm from an indifferent mass. Secondly it appears that the neural tube in the tail has entirely the colour of the anterior mark up to the very end. The caudal part of the spinal cord must, therefore, be formed from a part of the neural plate that is situated inside the area of the anterior mark. This observation, substantiated by numerous other observations, leads to the conception that after the end of the gastrulation the whole of the central nervous system is already projected on the neural plate. This, indeed, has also been communicated by Miss BIJTEL [1]).

The conception of BRACHET, who assumes that after the gastrulation notogenesis is still to take place, is no longer tenable now. The caudal part of the neural tube originates through intense stretching of the part of the neural plate (Fig. 1*a*), lying in the anterior mark. Later on nothing is added to this material, as the whole tailpart of the neural tube has in Fig. 1*e* the colour of the anterior mark. But it also appears from it that the caudal extremity of the neural tube must be situated inside the anterior mark, and that, therefore, the part of the neural plate, occupied by the central and the posterior mark does not partake in the formation of the central nervous system. What, then, has become of the last-named regions ?

It is certain that in stages such as are illustrated in Fig. 1*a* the invagination of material in the neighbourhood of the blastopore has not yet terminated. So, in the case described, the area of the posterior stained spot is still rolled inwards totally, or perhaps for the greater part, with the exception, of course, of that part of the mark that lay on the continuation of the neural folds, and that after the closure of the neural tube was to be found in the ectoderm-suture, from which part of the skin-ectoderm of the tail arose.

From the invaginated material have been formed the caudal part of the intestine, and some somites, viz. first the 16th somite of the trunk. It goes without saying, that this border is a matter of chance and depends on the extension of the central-mark. But the most caudal mesoderm segments show the stain of the central mark. They must have originated from the area of the neural plate which is indicated by stripes in Fig. 1*a* (central mark). It has already been stated that the caudal end of the neural tube

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[1]) J. H. BIJTEL: Nederl. Tijdschr. v. Geneesk. 72. I. 26. 1928.

is not situated near the blastopore, but a little more cranial to it. The part of the neural plate, occupied by the two posterior marks is, therefore, only in appearance a part of the neural plate. If, then, by the term neural plate is meant exclusively the anlage of the central nervous system (which would be quite reasonable) it should be borne in mind that the neural plate ends some way cranial to the blastopore, and that the neural folds also end there, although it would seem as if the folds are continued on either side of the blastopore.

In what way the posterior tail-myotomes have been developed could be fully ascertained in several cases. Change of situation and extension of stain marks, and comparison of conditions afterwards revealed in the experimental objects, enabled us to watch the fate of different cell-groups in the formation of the tail. It then appeared, that, a small part excepted, (in Fig. 1*a* the area of the posterior mark) the cell-material belonging apparently to the neural plate, which lies before the blastopore, (in Fig. 1*a* the area of the central mark), is not rolled inward round the rim of the blastopore, but, after the coalescence of the folds that encompass it, is found again in the interior of the tail-anlage, and adds new somites to those that have originated from the invaginated mesoderm-material.

In the investigation of the Axolotl-embryo, just discussed, the notochord appeared to be unstained, and to lie among the stained somites, as an unstained cell-strand. This can be understood only when assuming that the notochord did not originate from the same material as the somites of the tail; in other words that neither the material of the posterior mark, which is rolled inward, nor the area of the second mark has contained cellular material for the notochord in the tail. Then, probably, the notochord in the stage of Fig. 1*a*, as well as the tail part of the neural tube is already present in its entirety and proceeds (when the tail is formed) in the tail-anlage through stretching. From the fact that notochord, neural tube, and the greater part of the somites were of a different stain in the tail, it follows avowedly that they cannot have originated from one and the same indifferent cell-mass (the tail-bud).

It remains for us to say that by means of vital-dye marking in Rana esculenta the same development of the tail could be demonstrated as in Urodela, of which we discussed, for the sake of brevity only the Amblystoma, but of which Triton also yielded results in harmony with what has been said above. It only became evident, that in Rana esculenta more somites arose from the material round the blastopore than in Amblystoma, while the more cranial part of the neural plate, which no longer partakes of the invagination-movements, contributes less to the formation of the tail-mesoderm.

As Miss BIJTEL will soon discuss more at large all sorts of questions bearing on those treated briefly in this paper (conception of gastrulation, canalis neurentericus etc.) we shall no longer expatiate on the stainmarking experiments.

In 1927 WOERDEMAN had performed some experiments on Amblystoma-embryos, which also throw some light on the development of the tail, although they were carried out with a different object. But not before the results of the stain-marking experiments became known, could these experiments be fully explained. They corroborated those results. Hence we decided to report our results jointly.

Of Amblystoma embryos with clearly visible neural plate, as illustrated in Fig. 2, a large part of the neural plate was excised (outlined in Fig. 2). The incision was so deep that the gut-cavity was opened, and the material lying under the neural plate was also cut out. The whole rectangular, excised piece was then rotated through 180° (so that the caudal line of incision became cranial, and the cranial line caudal). After that it was grafted in the open wound, and made to coalesce again as well as possible in the rotated position. The caudal line of incision remained at some distance before the blastopore, and, as projected in Fig. 1*a*, will as a rule have lain in the area of the central mark, now slightly anteriorly, now again slightly posteriorly. The position of the anterior line is of less importance.

Fig. 2. Embryo of Amblystoma mexicanum. The outlined area was excised, rotated through 180° and then implanted again.

A number of animals operated upon now developed in a very peculiar manner. In Fig. 3 four of such animals have been represented.

They are all characterized by a rudimentary development of the tail, while the rotated part has also developed a tail, but this one grows cranially. Other particulars are observable, but they may be left out of consideration. On closer inspection it appears that the true tail of the animals consists chiefly of skin-ectoderm, some mesenchyme, and in a few cases also a little mass of axial mesoderm. In all these cases microscopical examination of the slice-series revealed that behind the posterior border of the grafted piece the neural tube and the notochord are lacking, but that myotomes occur. Their number differs. In the tail, however, that grows cranially from the rotated piece the normal structure can be observed in all cases up to the very end. There a neural tube, a notochord and axial mesoderm has developed. This is illustrated in Fig. 4. Fig. 4*a* is a section of the animal represented in Fig. 3*a* at the place marked by arrow 1, while Fig. 4*b* represents a section at the place marked by arrow 2. In Fig. 4*a* the anteriorly directed tail has been cut; (also the nervous system has just been cut, which has developed in the head from the anterior part of the neural plate). In this anlage of the tail we clearly see neural tube, notochord, and myotomes. But in the tail proper (Fig. 4*b*) only small myotomes and some mesenchyme are to be seen, but no neural tube and no notochord.

The animal represented in Fig. 3*b* has in the tail proper numerous myotomes, but no notochord, and no neural tube, as is shown in Fig. 5

which represents a section at the place, indicated by the arrow in Fig. 3*b*. As the parting organs between left and right myotomes (neural tube and

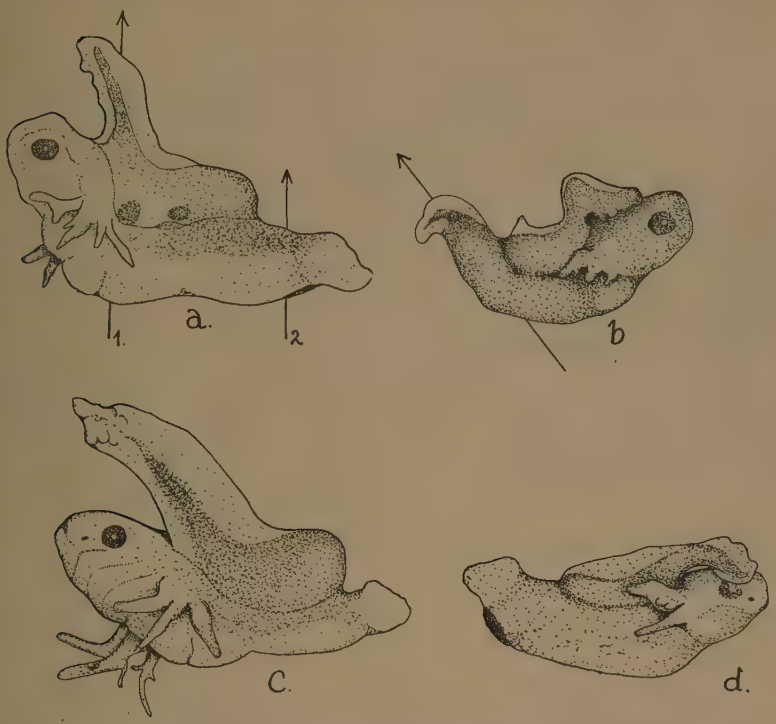

Fig. 3. Four embryos of Amblystoma mexicanum, Eleven days after operation according to Fig. 2. (Zeiss. Object $F = 55$, Oc. 2. $\times$ 3/4).

notochord) were wanting, in a number of places the muscle-fibres, which have been formed from the myotomes, have grown across the median line.

In explaining the result of the experiments it should be remembered that the caudal line of incision lay during the operation some way before the blastopore. Behind this line no neural tube nor a notochord has developed. The anlage of the two organs must, therefore, terminate caudally at some distance before this caudal line of incision.

This corresponds with the experience in the staining experiments: that the anlage of the central nervous system does not reach as far as the blastopore (in Fig. 1*a* the presumptive posterior end lies in the anterior mark). The material lying between the presumptive end of the anlage of the neural tube and the blastopore does not take part in the formation of the notochord of the tail, and the caudal part of the neural tube, otherwise the experimental animals, in which this material was not displaced, should

possess both organs in their true tail. On the contrary..... after the gastrulation has been completed the whole notochord and neural tube are

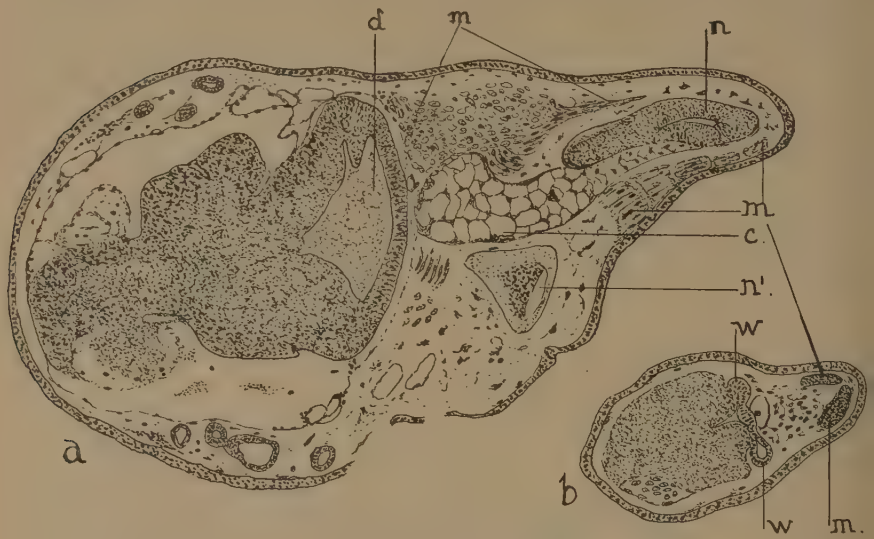

Fig. 4. Sections through the embryo represented in Fig. 3*a*.
Direction of the section of Fig. 4*a* is indicated in Fig. 3*a* by arrow 1
" " " " " " 4*b* " " " " " " " 2
(Protocol A II 1927, 1. VIII. 24 and 2. VII. 1) (Zeiss. Apochr. 16 mm Comp. Oc. K 4)
n = neural tube of tail *c* = notochord *d* = gut
n' = neural tube of head (just cut) *m* = myotomes *w* = WOLFFian duct in connection with gut.

present and they stretch intensely in their posterior parts for the formation of notochord, and spinal cord of the tail.

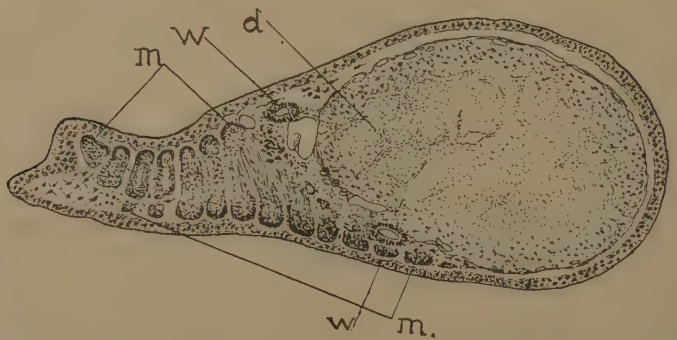

Fig. 5. Section through the embryo pictured in Fig. 3*b*. (Direction of the section is indicated in Fig. 3*b* by arrow).
(Protocol A. V. 1927, 2. II. 21.) (Zeiss. Aproch. 16 mm. Comp. Oc. K 4).

The capacity of stretching intensely appears distinctly in these Amblystoma-embryos, for the notochord and the neural plate extremities, rotated through 180°, are growing far beyond the anterior border of the rotated graft towards the head, and thus develop a tail in opposite direction.

Also the relations shown by the myotomes in the true tail, and in the one that grows anteriorly, are conceivable. If we assume that in a stage, such as is represented in Fig. 1*a*, a part of the neural plate was rotated, and the posterior incision went through the middle of the central mark, then after the rotation one part of this mark will be directed anteriorly, and another part will remain behind the incision. This latter part contributes together with the material lying round the blastopore to the formation of mesoderm for the true anlage of the tail. But notochord and neural tube cannot be formed from it. The part of the central mark placed anteriorly after the operation also supplies mesoderm for a tail, which, owing to the capacity of stretching of notochord and neural tube anlage, grows anteriorly.

Now, if we assume that in the embryo, pictured in Fig. 1*a*, the posterior border of the piece to be rotated, is lying close to the posterior mark, so that nearly the whole of the central mark is rotated along with it, then a condition will arise as in Fig. 3*c*, in which behind the posterior border of the rotated graft a very small piece of the trunk is to be seen, and a very rudimentary tail. But the tail directed anteriorly is well developed

If, on the contrary, the posterior border of the graft should lie closer to the anterior mark, the central mark would for the greater part keep its own place, and the larva would receive a smaller quantum of mesoderm in its anteriorly directed tail, and a larger one in its true tail (Fig. 3*a* and 3*b*).

In all experiments in which parts of the dorsal half of the embryo were rotated, the blastopore and a rather large area before it, kept its own place. Part of the material, which lies between the posterior extremity of the neural anlage and the blastopore, and which according to the stainmarking experiments supplies tail-mesoderm, takes up an anterior position after rotation of the graft through 180°, and now forms mesoderm for the tail that grows in a cranial direction. This proves at the same time that it is not necessary for this presumptive mesoderm to be carried in with the invagination in the process of gastrulation. It should be remembered that the staining-experiments also led to the opinion that part of this presumptive mesoderm in case of normal development was not invaginated.

Finally let it be pointed out that the true tail, as well as the rotated one, exhibits a more or less distinct tail-fin. The skin for this fin has originated with the coalescence of the neural folds. Also the tail proper has this fin-formation, for at the operation the caudal continuations of the neural folds have retained their place.

Thus it is evident, that the rotation-experiments are in perfect harmony with the results of the staining experiments and in large measure lend support to the conceptions based on the staining experiments.

Summary.

1. The conception that the tail arises from an indifferent cell-mass (tailbud) which, while growing caudally, gives origin to the different organs of the tail through differentiation, is not in keeping with our experience.

2. The notochord of the tail and the caudal portion of the neural tube are totally present in stages with slitlike blastopore, and without tail-bud, but take their final shape through intense stretching of their anlage. In this process, however, no new material is assimilated.

3. The posterior part of the anlage of the notochord and the anlage of the central nervous system lies at some distance in front of the blastopore, so that between the blastopore and the posterior extremity of the neural anlage (which is not visible outwardly), an area is left, of which the material appears to provide mesoderm for the tail.

4. To this end the material lying in the environment of the slitlike blastopore is first carried inward round the rims of the blastopore (continuation of the cell-displacements in gastrulation). It contributes to the formation of the hindmost trunk-segments and somites of the tail-root, together with non-segmented mesoderm of the same region.

5. The presumptive mesoderm that lies more cranial from the blastopore is not concerned in this invagination. It provides the posterior somites of the tail.

6. Cell-groups that lie ventrally and laterally to the blastopore (in stages with slitlike blastopore) and which are carried inward, contain the anlage of the tail gut.

7. The ectoderm for the skin of the tail lies in stages with slitlike blastopore partly quite ready on the neural folds and on their caudal continuation on each side of the blastopore. Part of the skin of the tail is supplied through stretching of this ectoderm of the folds.

8. The folds just alluded to also supply mesenchyme for the tail.

From the Anatomical and Embryological Institute of the State-University of Groningen.

Physics. — *On the anomalous g-values in the spectrum of ionized Argon (Ar II).* By C. J. BAKKER. (Communicated by Prof. P. ZEEMAN).

(Communicated at the meeting of October 27, 1928).

1. *Introduction.*

In a former paper in these Proceedings [1]) the investigation of the ZEEMAN-effect of 110 lines of the spectrum of ionized Argon (*Ar II*) has been communicated. It has appeared that many terms, which are known by the extensive analysis of the spectrum by DE BRUIN [2]), possess g-values deviating from the well known splitting factors of LANDÉ. For the terms arising from the $4s$, $4p$ and $5s$ electron the g-sum rule could be tested.

The g-formula of LANDÉ is derived on the assumption that the coupling scheme of RUSSELL-SAUNDERS [3]), which occurs mostly in the spectra, holds. Assumption of other coupling schemes gives rise by calculation to g-values deviating from those of LANDÉ. This communication contains the calculation of the g-values belonging to 4 different coupling schemes and the comparison of the g-values calculated in that way with those found experimentally in the spectrum of ionized Argon (*Ar II*) for the coupling $4s$, $4p$, $5s$ (^3P).

2. *Method of calculation of the g-values.*

According to the theory of HEISENBERG, PAULI and HUND the energy levels of the spectrum of the simply ionized Argon atom (*Ar II*) can be deduced by joining to the double ionized Argon atom (*Ar III*), that is the atomic core of Argon *II*, in succession a $3p$, $3d$, $4s$, $4p$, $4d$, $5s$..... electron and by coupling the quantum vectors of core and electron [4]).

We denote by

s_1 = rotation quantum number of the atomic core,

l_1 = azimuthal quantum number of the atomic core.

and

s_2 = rotation quantum number of the electron that is coupled.

l_2 = azimuthal quantum number of the electron that is coupled.

[1]) C. J. BAKKER, T. L. DE BRUIN and P. ZEEMAN: These Proceedings **31**, 780, 1928, Zeitschr. für Phys. **51**, 114, 1928. **52**, 299, 1928.

[2]) T. L. DE BRUIN: These Proceedings **31**, 591, 1928 and **31**, 771, 1928. Zeitschr. für Phys. **48**, 62, 1928 and **51**, 108, 1928.

[3]) H. N. RUSSELL and F. A. SAUNDERS: Astrophys. Journal **61**, 38, 1925. Also F. HUND: Zeitschr. für Phys. **33**, 345, 1925 and S. GOUDSMIT: Zeitschr. für Phys. **32**, 794, 1925.

[4]) Resumed description by F. HUND, Linienspektren, Springer, Berlin, 1927.

From GOUDSMIT and UHLENBECK [1]) I take some coupling schemes and the method of calculation of the g-values for these schemes. To draw up the g-formulas one makes use consequently of a condition given by LANDÉ [2]), which thus runs: When a vector z is composed of two vectors x and y whereby

$$|x-y| \leqq z \leqq x+y$$

then the splitting factor $g(z)$ is determined by:

$$g(z) = \frac{z(z+1)+x(x+1)-y(y+1)}{2z(z+1)} g(x) + \frac{z(z+1)+y(y+1)-x(x+1)}{2z(z+1)} g(y).$$

As given is supposed, that

$$g(l_1) = g(l_2) = 1.$$

and

$$g(s_1) = g(s_2) = 2$$

Scheme I. $\{(s_1 l_1)(s_2 l_2)\} = (j_1 j_2) = j.$

The reciprocal influence of core and electron is so small, that the couplings of s_1 with l_1 (both of the core) and of s_2 with l_2 (both of the electron) is not disturbed. The external magnetic field is not so strong that the coupling of j_1 and j_2 is disturbed.

The g-formula becomes

$$g(j) = \frac{j(j+1)+j_1(j_1+1)-j_2(j_2+1)}{2j(j+1)} g(j_1) + \frac{j(j+1)+j_2(j_2+1)-j_1(j_1+1)}{2j(j+1)} g(j_2).$$

in which is

$$g(j_1) = \frac{j_1(j_1+1)+s_1(s_1+1)-l_1(l_1+1)}{2j_1(j_1+1)} \cdot 2 + \frac{j_1(j_1+1)+l_1(l_1+1)-s_1(s_1+1)}{2j_1(j_1+1)} \cdot 1$$

and

$$g(j_2) = \frac{j_2(j_2+1)+s_2(s_2+1)-l_2(l_2+1)}{2j_2(j_2+1)} \cdot 2 + \frac{j_2(j_2+1)+l_2(l_2+1)-s_2(s_2+1)}{2j_2(j_2+1)} \cdot 1.$$

Scheme II. $[\{(s_1 l_1) s_2\} l_2] = \{(j_1 s_2) l_2\} = (s' l_2) = j.$

By means of the reciprocal influence of core and electron the coupling between s_2 and l_2 is now broken. The s_2 of the electron is coupled with the j_1 (resultant of s_1 and l_1) of the core, which gives a resultant s'. s' is coupled with l_2, this gives the total resultant j.

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[1]) S. GOUDSMIT and G. E. UHLENBECK: Zeitschr. für Phys. **35**, 618, 1926.
S. GOUDSMIT and E. BACK: Zeitschr. für Phys. **40**, 530, 1927.
S. GOUDSMIT: Physica **5**, 419, 1925.

[2]) E. BACK and A. LANDÉ: ZEEMAN-effekt und Multipletstruktur, Springer, Berlin, 1925.

The g-values are calculated from

$$g(j)=\frac{j(j+1)+s'(s'+1)-l_2(l_2+1)}{2j(j+1)}g(s')+\frac{j(j+1)+l_2(l_2+1)-s'(s'+1)}{2j(j+1)}\cdot 1.$$

in which

$$g(s')=\frac{s'(s'+1)+j_1(j_1+1)-s_2(s_2+1)}{2s'(s'+1)}g(j_1)+\frac{s'(s'+1)+s_2(s_2+1)-j_1(j_1+1)}{2s'(s'+1)}\cdot 2$$

in which $g(j_1)$ is calculated as in scheme I.

Scheme III. $[\{(s_1\, l_1)\, l_2\}\, s_2]=\{(j_1\, l_2)\, s_1\}=(l'\, s_2)=j.$

Now the l_2 of the electron is coupled with the j_1 of the core, which gives a resultant l'. l' coupled with s_2 gives the total resultant j.

The g - formula is

$$g(j)=\frac{j(j+1)+l'(l'+1)-s_2(s_2+1)}{2j(j+1)}g(l')+\frac{j(j+1)+s_2(s_2+1)-l'(l'+1)}{2j(j+1)}\cdot 2.$$

in which is

$$g(l')=\frac{l'(l'+1)+j_1(j_1+1)-l_2(l_2+1)}{2l'(l'+1)}g(j_1)+\frac{l'(l'+1)+l_2(l_2+1)-j_1(j_1+1)}{2l'(l'+1)}\cdot 1.$$

in which $g(j_1)$ is calculated as in scheme I.

Scheme IV. $\{(s_1\, s_2)\,(l_1\, l_2)\}=(s\, l)=j.$

This is the coupling scheme of RUSSELL and SAUNDERS, also the most frequently occurring. The reciprocal influence of core and electron is large, so that s_1 is coupled with s_2 and l_1 with l_2. The external magnetic field is not so strong, that the coupling of s and l is disturbed.

The g - values are calculated from

$$g(j)=\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}g(s)+\frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)}g(l)$$

in which

$$g(s)=\frac{s(s+1)+s_1(s_1+1)-s_2(s_2+1)}{2s(s+1)}\cdot 2+\frac{s(s+1)+s_2(s_2+1)-s_1(s_1+1)}{2s(s+1)}\cdot 2=2$$

and analogical

$$g(l)=1.$$

One thus finds here the g-formula of LANDÉ

$$g(j)=1+\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}.$$

3. *Application to Ar II.*

The energy levels of the simply ionized Argon atom (*Ar II*), of which the g - values are determined experimentally, are levels built upon the stable ground state of the double ionized atom (*Ar III*). This is a ^3P state.

The quantum numbers s_1 and l_1 are inherent to this ^3P state. With the assumption, that for this lowest term in the energy scheme of double ionized Argon the coupling of the quantum vectors is the normal one, according to scheme IV, one must give the quantum numbers the values

$s_1 = 1$ (gives account for the triplet state) and

$l_1 = 1$ (gives account for the P state).

If a $s =$ electron is coupled, then: $s_2 = {}^1/_2$, $l_2 = 0$.

„ „ $p =$ electron „ „ „ $s_2 = {}^1/_2$, $l_2 = 1$ and so on.

It may be supposed well kwown that the different coupling schemes give with each choice of $s_1\, l_1$ and $s_2\, l_2$ the same number of terms with the same quantum numbers j. [1])

TABLE 1. Terms of the 4*p*-electron.

j	Scheme I		Scheme II		Scheme III		Scheme IV	g-values				
	j_1	j_2	s'	l_2	l'	s_2		Scheme I	Scheme II	Scheme III	Scheme IV	Observed
$^5/_2$	2	$^3/_2$	$^5/_2$	1	2	$^1/_2$	$^4P_3$	1.44	1.53	1.53	1.60	1.60
$^3/_2$	2	$^3/_2$	$^3/_2$	1	1	$^1/_2$	$^4P_2$	1.47	1.49	1.50	1.73	1.73
$^1/_2$	1	$^1/_2$	$^3/_2$	1	0	$^1/_2$	$^4P_1$	1.78	2.11	2.00	2.67	2.67
$^7/_2$	2	$^3/_2$	$^5/_2$	1	3	$^1/_2$	$^4D_4$	1.43	1.43	1.43	1.43	1.43
$^5/_2$	1	$^3/_2$	$^3/_2$	1	2	$^1/_2$	$^4D_3$	1.40	1.40	1.40	1.37	1.33
$^3/_2$	0	$^3/_2$	$^3/_2$	1	2	$^1/_2$	$^4D_2$	1.33	1.29	1.30	1.20	1.20
$^1/_2$	0	$^1/_2$	$^1/_2$	1	1	$^1/_2$	$^4D_1$	0.67	0.67	0.67	0 00	0.00
$^3/_2$	2	$^1/_2$	$^5/_2$	1	1	$^1/_2$	$^4S_2$	1.67	1.84	1.83	2.00	2.00
$^5/_2$	2	$^1/_2$	$^3/_2$	1	3	$^1/_2$	$^2D_3$	1.33	1.24	1.24	1.20	1.24
$^3/_2$	1	$^1/_2$	$^1/_2$	1	2	$^1/_2$	$^2D_2$	1.22	1.11	1.10	0.80	0.90
$^1/_2$	1	$^3/_2$	$^1/_2$	1	1	$^1/_2$	$^2P_1$	1.22	0.89	1.00	0.67	0.99
$^3/_2$	1	$^3/_2$	$^1/_2$	1	1	$^1/_2$	$^2P_2$	1.38	1.33	1.33	1.33	1.23
$^1/_2$	2	$^3/_2$	$^3/_2$	1	1	$^1/_2$	$^2S_1$	1.67	1.67	1.67	2.00	1.68

[1]) S. GOUDSMIT and G. E. ÜHLENBECK: Zeitschr. für Phys. **35**, 618, 1926. R. DE L. KRONIG: Proc. Nat. Ac. of Sc. **12**, 330, 1926.

TABLE 2. Terms of the 5*s*-electron.

j	Scheme I		Scheme II		Scheme III		Scheme IV	g-values				
	j_1	j_2	s'	l_2	l'	s_2		Scheme I	Scheme II	Scheme III	Scheme IV	Observed
$^5/_2$	2	$^1/_2$	$^5/_2$	0	2	$^1/_2$	$^4P_3$	1.60	1.60	1.60	1.60	1.60
$^3/_2$	2	$^1/_2$	$^3/_2$	0	2	$^1/_2$	$^4P_2$	1.40	1.40	1.40	1.73	1.63
$^1/_2$	0	$^1/_2$	$^1/_2$	0	0	$^1/_2$	$^4P_1$	2.00	2.00	2.00	2.67	2.53
$^3/_2$	1	$^1/_2$	$^3/_2$	0	1	$^1/_2$	$^2P_2$	1.67	1.67	1.67	1.33	1.43
$^1/_2$	1	$^1/_2$	$^1/_2$	0	1	$^1/_2$	$^2P_1$	1.33	1.33	1.33	0.67	0.81

Table 1 contains the observed and calculated g-values according to the 4 schemes for the terms, arising from the coupling of the 4 p-electron. Table 2 gives the same for the terms of the 5 s-electron.[1]) For so far terms with the same inner quantum number are mentioned there is a difficulty as to the manner in which one must adjoin the terms of the schemes I, II and III to those of scheme IV. In the tables the g-values of the schemes I, II and III are as much as possible taken in accordance with those of scheme IV[2]). In table 2 the g-values 1.40 and 1.67 of the terms with $j = {}^3/_2$ are for example yet changeable. In connection with the observed g-values the given allotment seems the most probable.

As is to be expected on account of theoretical considerations, the tables show, that for the terms of the 4 p and 5 s-electron none of the coupling schemes holds exactly.

4. *Summary.*

In connection with the anomalous g-values, that appear in the spectrum of the simply ionized Argon atom (*Ar II*), g-values are calculated according to 4 different coupling schemes for the terms arising from the coupling of the 4 p and 5 s-electron.

Laboratory "Physica" University of Amsterdam.

October 1928.

[1]) The g-values of the terms of the 4s-electron agree exactly with those of LANDÉ (scheme IV).

[1]) S. GOUDSMIT: Thesis, Leiden, 1927.
F. HUND: Zeitschr. für Phys. **34**, 296, 1925.

Physics. — *Measurement of the thermo-electric* THOMSON *effect down to the temperature of liquid hydrogen.* By G. BORELIUS, W. H. KEESOM and C. H. JOHANSSON. (Comm. N°. 196a from the Physical Laboratory at Leiden.)

(Communicated at the meeting of November 24, 1928).

§ 1. *Introduction.* Earlier measurements of the THOMSON effect down to the temperature of liquid air have given no definite knowledge concerning the temperature dependence of the effect. As however, the knowledge concerning this temperature dependence is of great importance with regard to the electron theory of metals we resolved to proceed to lower temperatures.

The present investigation embraces measurements of two wires of copper and silver, alloyed with small quantities of gold, down to 20° K., and further, measurements of pure copper down to 70° K. The composition of the alloyed wires had to be chosen with respect to the possibility of performing the measurements at the lowest temperatures. The measurement of these wires (of which a sufficient quantity is being retained as thermo-electric normals) however afterwards enable the determination of the THOMSON effect for pure metals or other alloys, which may prove of special interest, by means of measuring the thermo-electric force of these metals against one of the wires investigated here. As a matter of fact, the thermo-dynamic theory of THOMSON gives the relation

$$\sigma_1 - \sigma_2 = T \frac{de}{dT}, \quad \ldots \ldots \ldots \quad (1)$$

where e is the thermo-electric force per degree of two metals with the THOMSON effect σ_1 and σ_2 at the absolute temperature T.

By means of measuring the THOMSON effect of two wires to the temperature of liquid hydrogen, we have further been able by the aid of formula (1) to obtain a control of our measurements of this effect after having measured also the thermo-electric force of a couple formed by these wires.

§ 2. *The measuring method* was on the whole the same as the one used earlier by BORELIUS [1]) and by BORELIUS and GUNNESON [2]). A wire of the metal, which was to be investigated, was so arranged that the ends were kept at a constant temperature. An electric current sent through the

[1]) G. BORELIUS. Ann. d. Phys. (4) **63**, 845, 1920.

[2]) G. BORELIUS and F. GUNNESON. Ann. d. Phys. (4) **65**, 520, 1921.

wire causes a rise in its temperature which is greatest in the middle part of the wire. In one part of the wire the current then goes to a higher temperature, in the other part towards a lower one. The THOMSON effect in the two parts of the wire occasions heat actions of different signs, which change when the direction of the current is changed. The THOMSON effect may be obtained in volt/degree from the formula

$$\sigma = \frac{\lambda}{I} \frac{\tau}{t} \frac{3q}{y} \Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where y is the distance in cm between a certain point on the wire and its middle point, t the increase in temperature at the point considered caused by the current heat, τ the change in temperature caused by the THOMSON effect at the same point, q the cross section of the wire in cm^2, λ the spec. heat-conductivity in watt/cm degree, I the current intensity in ampères and Φ a correction factor, the value of which at a suitable choice of wire dimensions and current intensity is situated near 1. The mathematical formula of Φ is to be found in the earlier publications referred to.

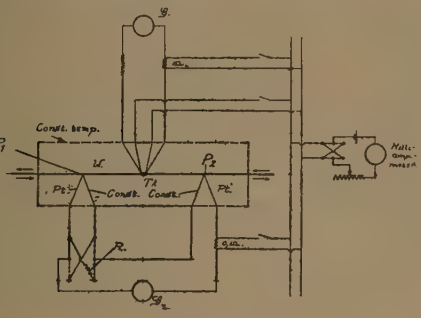

Fig. 1.

The connection diagram, which in several respects differs from the ones earlier used, is seen in Fig. 1. τ and t have been measured with thermo-elements at two symmetrically situated points P_1 and P_2 of the wire W. In the measurement of t the thermo-elements were coupled in series; in the measurement of τ against each other. In the determination of t the current was alternately closed and opened; in the determination of τ it was repeatedly reversed. The same galvanometer G_z was used for both determinations, a supersensitive ZERNIKE galvanometer, the sensitivity of which was considerably lowered when the thermo-elements were coupled in series, by the automatic attaching of a big resistance of 2000—10000 ohm. The joint resistance of the galvanometer and the thermo-elements amounted to about 50 ohm. The sensitivity of the galvanometer was estimated in the different cases by sending known currents, measured with a milliampèremeter, through a resistance of 0.1 ohm, which was constantly coupled to the galvanometer circuit. The determination of the temperature of the wire was made by means of a platinum thermometer Th, wound around its middle point.

The experimental arrangement is illustrated by Fig. 2. In order to obtain a constant surrounding temperature, the measuring wire W was attached to a block of copper B, and covered by a radiation screen of copper Sc. The block was cooled by direct contact of the cooling liquids L

which were supplied to a receptacle R of capacity about 0.4 liter by means of a thin electro-plated tube T. The block and the receptacle were sur-

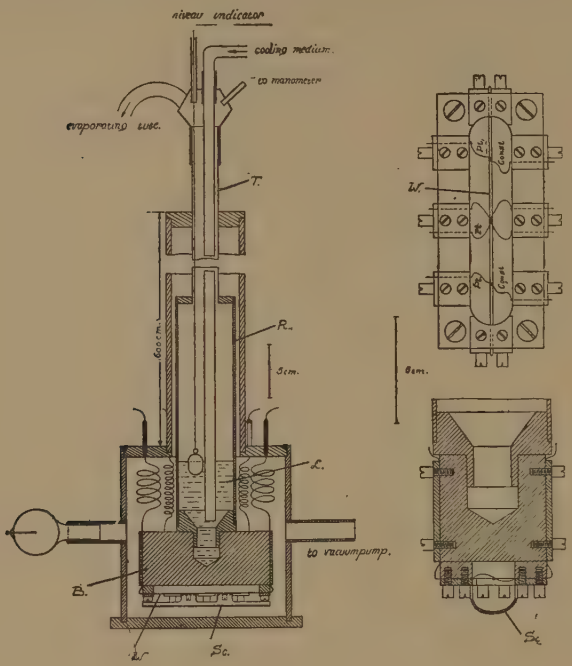

Fig. 2.

rounded by vacuo. At a good vacuum and with one hydrogen filling measurements could be made during about 80 min. The heat capacity of the block — which weighed 3 kg — was enough to enable the temperature, when the cooling medium had evaporated, to rise sufficiently slowly to allow of measurements even during the rise in temperature. The measuring wire and the eight leads (0.25 mm copper wires) to the thermo-elements and to the resistance thermometer were fastened to the block with a good electric insulation, and with the best possible heat conducting contact, for which purpose thin cigarette paper was put to a convenient use as interlining. Further the measuring wires, which were about 1 mm thick, were covered by a thin coating of enamel lacquer which enabled a direct placing of the resistance thermometer and the thermo-elements on the wire, where they were fixed by means of shellac. We are indebted to Mr J. LAGERKVIST, of the Sieverts Kabelverk firm, Stockholm, for rendering kind assistance with the coating of the wires.

The resistance thermometers consisted of 0.05 mm physically pure platinum from the HERAEUS firm. They were wound about 20 times round

the measuring wire, their resistance being about 4 ohm at room temperature. Each thermometer was aged, after having been applied on the wire, by successive coolings and heatings with a subsequent calibration in the apparatus at a number of temperatures obtained by means of melting ice, liquid hydrogen, and nitrogen or oxygen, boiling at different pressures.

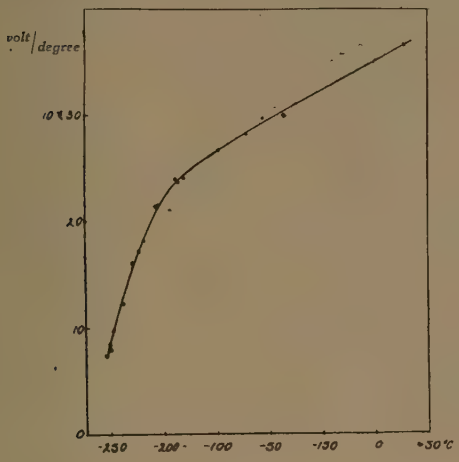

Fig. 3.

The thermo-elements consisted of 0.05 mm platinum and constantan wires. Fig. 3 represents the thermo-electric force per degree for one of these thermo-elements, determined by means of comparison with the resistance thermometer. It is not necessary, in order to estimate the THOMSON heat according to formula (2) to know the thermo-electric force very accurately, as this term only comprises the ratio between the quantities τ and t, measured by one and the same thermo-element. As however the temperature differences τ and t are situated at somewhat varying mean temperatures, the ratio between them must not be considered equal to the ratio between the measured thermo-electric forces, but a slight correction must be introduced for the relative change in the thermo-electric force between these mean temperatures. This correction was considerable only at the lowest temperatures and amounted, in the worst case, to 12 per cent.

The correction factor Φ in formula (2) is introduced on account of the resistance of the measuring wire changing with the temperature, and the external heat conductivity. As in the worst cases Φ deviated from 1 with only —2.5 and + 1.8 per cent respectively, it was quite sufficient to use the temperature coefficient from earlier measurements of the resistance and to estimate the radiation from the black-coated surface of the wire as 87 per cent of black body radiation.

Formula (2) further comprises the specific heat conductivity. For the wires investigated at liquid hydrogen temperatures this was determined at two temperatures according to LEES' method. One end of a piece of the wire was fastened to the copper block while a known amount of heat was supplied electrically to the other end. The decrease in temperature in the wire was then measured by means of two resistance thermometers, at a distance of about 6 cm. The results of the heat conductivity measurements are given in Table I. These values have been noted down together with

values of other authors in T/λ, T-diagrams figs 4 and 5 (full curves). They very much resemble resistance-temperature-diagrams, and are especially well suitable for a graphical determination of λ. The curves obtained for λ have

TABLE I.

Heat conductivity.					
Copperalloy			Silveralloy		
T °K	λ Watt / cm. degree	$\frac{T}{\lambda}$	T °K	λ Watt / cm. degree	$\frac{T}{\lambda}$
18.8	1.44	13.0	18.9	2.79	6.77
78.9	2.87	27.5	93.4	4.22	22.15

also been traced in the figures mentioned (broken curves). For the technically pure copper wire examined by ourselves only as far as to the

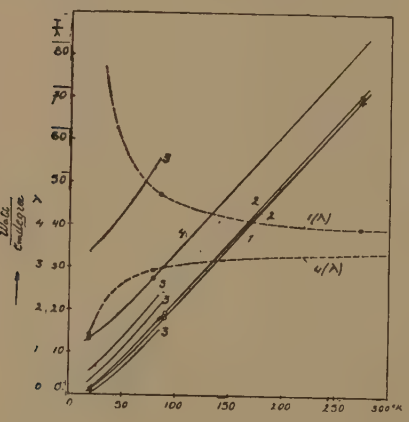

Fig. 4.

$\frac{T}{\lambda}$ (and λ) for copper.

1. R. SCHOTT, technically pure,
2. W. MEISSNER [1]), very pure,
3. GRÜNEISEN and GOENS [2]), pure and alloyed,
4. Copper with 0.37 at. % Au.

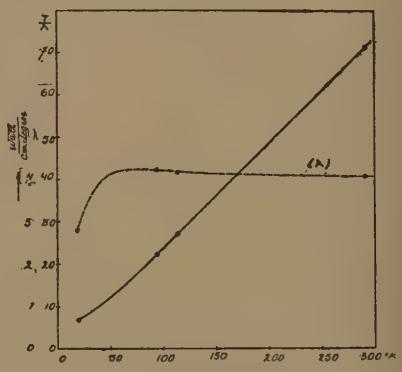

Fig. 5.

$\frac{T}{\lambda}$ (and λ) for silver.

—•— silver with 0.37 at. % Au

—○— CH. H. LEES [3]), 99.9 weight % Ag.

[1]) W. MEISSNER. Ann. d. Phys. (4) **47**, p. 1001, 1915.

[2]) E. GRÜNEISEN and E. GOENS. Zs. f. Phys. **44**, p. 615, 1927.

[3]) CH. H. LEES. Phil. Trans. A **208**, p. 381, 1908.

temperature of liquid air, the values obtained by SCHOTT [1]) for similar substances have been applied.

In reality formula (2) presupposes that λ is independent of the temperature, which applies approximately for pure metals at higher temperatures. At lower temperatures, however, the heat conductivity of pure metals increases with decreasing temperature so considerably that formula (2) may scarcely be applied with a sufficient degree of approximation. Further, the temperature change t decreases with the increase in heat conductivity and the simultaneous decrease in the resistance. This makes the measuring of pure metals at the lowest temperatures too uncertain. We have therefore used pure copper only down to the temperature of liquid nitrogen, for the lower temperatures using as measuring wire copper and silver wires respectively, alloyed with gold.

Our purpose was to add so much gold that it would make λ as little as possible dependent upon the temperature, which we calculated to attain by the addition in both cases of 0.37 atomic per cent of gold. Probably on account of another impurity present at the same time, the influence of the admixture was greater than we had anticipated, and the heat conductivity of the investigated wires decreased to an inappropriate degree at the lowest temperatures. A contributing factor for the copper wire may moreover have been the fact that it was investigated in a hard-drawn condition.

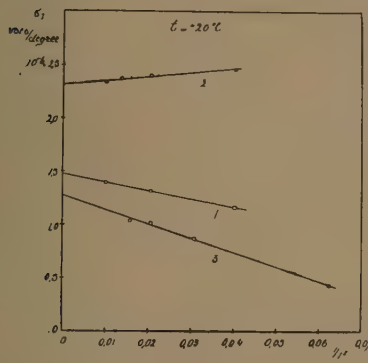

Fig. 6.
1. copper.
2. copperalloy.
3. silveralloy.

A very inconvenient source of error at all measurements of the THOMSON effect is the PELTIER influence caused by slight inhomogeneities in the wire. These PELTIER influences were also clearly evidenced in our measurements by the fact that the apparent THOMSON effect directly calculated according to formula (2), in general was dependent on the current intensity I. It is difficult to decide whether this influence emanates from inhomogeneities in the original wire or from differences between the fastened and the free parts of the wire. As this source of error is predominating, the actual THOMSON effect σ may however be calculated from the apparent one σ_I according to the formula

$$\sigma = \sigma_I + const. \frac{1}{I^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which, the constant being unknown, presupposes measurements with at least two current intensities at the same temperature.

[1]) R. SCHOTT. Verh. D. phys. Ges. 18, 27, 1916.

Accordingly, within each temperature range measurements have been made with at least two different intensities of current. In several cases more than two intensities have been used whereby within the limits of the accidental errors formula (3) was found to be confirmed. Fig. 6 gives examples of these measurements.

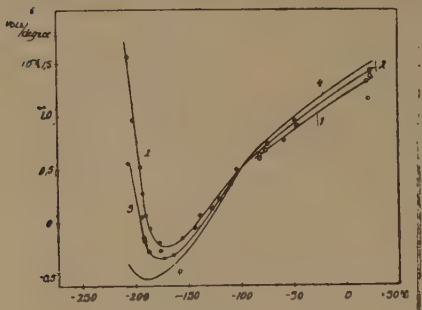

Fig. 7.

THOMSON-effect of copper.
1. 7 amp. 3. 14 amp.
2. 10 „ 4. corrected (to $\frac{1}{I^2} = 0$).

§ 3. *The results.* The direct results of the measurements are given in Table II and Figs 7—9. As temperature at which the measured THOMSON effect refers is taken the mean value of the temperatures of the resistance thermometer with a current and without current in the measuring wire.

The THOMSON effects, graphically determined according to formula (3), are compiled in Table III and Fig. 10.

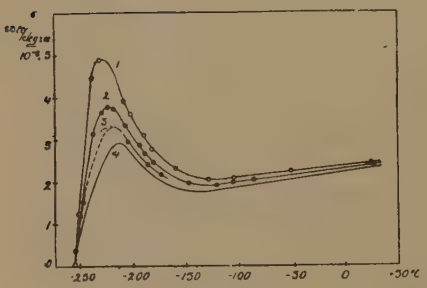

Fig. 8.

THOMSON-effect of copperalloy.
1. 7 amp. 3. 14 amp.
2. 10 „ 4. corrected.

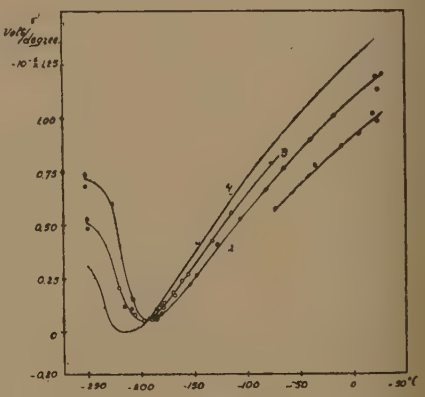

Fig. 9.

THOMSON-effect of silveralloy.
1. 7 amp. 3. 14 amp.
2. 10 „ 4. corrected.

§ 4. *Measurements of thermo-electric forces.* In order to check our measurements of the THOMSON effect with the aid of formula (1) as well as to enable certain conclusions concerning the THOMSON effects at still lower temperatures, we have further measured the thermo-electromotive forces of couples formed by the wires investigated. One of the junctions was accordingly kept at 0° C., the other at varying temperatures, two within

TABLE II.

Copper		Copperalloy		Silveralloy	
t °C.	$\sigma_I . 10^6$	t °C.	$\sigma_I . 10^6$	t °C.	$\sigma_I . 10^6$
7 amp.		7 amp.		7 amp.	
− 193	+ 0.876	− 255.2	+ 0.037	− 73.5	+ 0.578
81.5	0.594	251.7	1.25	36	0.780
76.5	0.607	238.9	4.47	10.5	0.867
58.5	0.767	230.3	4.90	+ 19	1.02
46.5	0.903	208.5	3.93	22.5	0.988
+ 20.5	1.320	202	3.62	23.5	0.986
22	1.160	189.5	3.12		
24	1.374	182	2.79		
		159.5	2.32	10 amp.	
		129	2.05	− 252	+ 0.682
10 amp.		105	2.08	252	0.739
− 206.5	+ 1.56	51	2.24	227	0.601
202.5	0.961	+ 25	2.43	208	0.151
195.3	0.520			192.5	0.062
193	0.272			188.5	0.071
190	0.061	10 amp		187.5	0.061
186	− 0.057	− 254.2	+ 0.36	185.5	0.058
180	− 0.269	250.0	1.20	185	0.071
177	− 0.189	237.3	3.15	181	0.085
155.5	− 0.155	229.2	3.66	154	0.220
139	+ 0.062	223.7	3.79	147.5	0.265
120.5	0.216	218.6	3.74	128	0.410
104	0.498	207	3.35	106.5	0.526
81.5	0.628	193.5	2.89	81.5	0.663
74	0.736	188.5	2.68	65	0.766
48	0.948	180.5	2.47	40	0.899
+ 23.5	1.427	173.5	2.18	18	1.009
23.5	1.410	147.5	1.97	+ 21.5	1.193
		121	1.91	23.5	1.133
		105.5	1.97	28	1.204
14 amp.		86	2.03		
− 206.3	+ 0.558	+ 27.5	2.38		
194.5	0.050			14 amp.	
192.5	− 0.148			− 250.6	+ 0.530
191.5	− 0.181	14 amp.		250.0	0.485
187.5	− 0.280	− 251.1	+ 0.79	221	0.206
173	− 0.341	247.0	1.53	216	0.119
164.5	− 0.312	204.5	2.96	209.5	0.109
144	− 0.055	186	2.42	206	0.080
127.5	+ 0.125			198	0.051
109	0.357			190.5	0.054
				186	0.106
				182	0.124
				181	0.112
				179.5	0.112
				178.5	0.137
				170	0.187
				168.5	0.171
				162	0.241
				156	0.270
				133	0.426
				115	0.559
				77	0.790

TABLE III.

T °K	Copper		Copperalloy		Silveralloy	
	$\sigma.10^6$	$\sigma/T.10^8$	$\sigma.10^6$	$\sigma/T.10^8$	$\sigma.10^6$	$\sigma/T.10^8$
20	—	—	+ 0.44	+ 2.20	+ 0.31	+ 1.55
30	—	—	1.39	4.63	0.24	0.80
40	—	—	2.17	5.43	0.10	0.25
50	—	—	2.84	5.68	— 0.02	— 0.04
60	—	—	2.96	4.93	— 0.02	— 0.03
70	— 0.26	— 0.37	2.71	3.87	+ 0.02	+ 0.03
80	— 0.46	— 0.57_5	2.45	3.06	0.08	0.10
90	— 0.49	— 0.54_5	2.20	2.45	0.14	0.15_5
100	— 0.45	— 0.45	2.00	2.00	0.21	0.21
110	— 0.37	— 0.33_5	1.85	1.68	0.28	0.25_5
120	— 0.26	— 0.21_5	1.75	1.46	0.36	0.30
130	— 0.12	— 0.09	1.71	1.31_5	0.43	0.33
140	+ 0.02	+ 0.01_5	1.73	1.23_5	0.50	0.36
150	0.16	0.10_5	1.77	1.18	0.57	0.38
160	0.32	0.20	1.80	1.12_5	0.63	0.39_5
170	0.47	0.25_5	1.84	1.08	0.70	0.41
180	0.59	0.33	1.88	1.04_5	0.76	0.42_5
200	0.80	0.40	1.95	0.97_5	0.87	0.43_5
220	0.96	0.43_5	2.03	0.92	0.98	0.44_5
240	1.10	0.46	2.11	0.88	1.08	0.45
260	1.24	0.47_5	2.18	0.84	1.18	0.45_5
280	1.38	0.49	2.26	0.81	1.28	0.45_5
300	1.52	0.50_5	2.33	0.78	1.38	0.46

the range of liquid hydrogen, two within the range of liquid nitrogen, and one at room temperature. In these measurements we have also included lead, which is often used as thermo-electric normal. The results are collected in Table IV. A positive sign indicates that the thermo-electric current in the junction which is kept at 0° proceeds towards the copper alloy.

By means of combining again and again two neighbouring temperatures

the thermo-electric forces per degree were calculated. They are given in Table V.

These values may be used as a control of the measurements of the

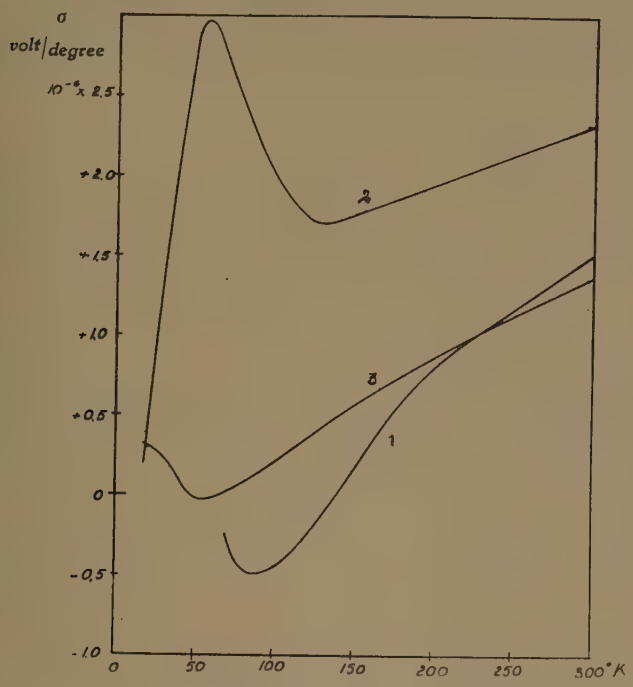

Fig. 10. THOMSON-effect.
1. copper. 2. copperalloy. 3. silveralloy.

TABLE IV.

E. M. F. against copperalloy (microvolt)			
°K	Silveralloy	Copper	Lead
14.22	— 435.95	— 581.9	— 26.1
20.36	— 406.85	— 552.5	— 3.1
63.63	— 234.8	— 350.05	+ 121.1
77.58	— 199.1	— 301.35	+ 139.4
273.09	0	0	0
288.29	+ 5.7	+ 13.16	— 32.0

TABLE V.

Thermo-electric Force per degree against copperalloy (microvolt/degree)			
°K	Silveralloy	Copper	Lead
17.3	+ 4.74	+ 4.80	+ 3.75
70.6	+ 2.56	+ 3.49	+ 1.31
280.7	+ 0.37	+ 0.87	− 2.11

THOMSON effects. As a matter of fact, according to formula (1) the change in thermo-electric force between two temperatures T' and T'' is

$$e''-e'=\int_{T'}^{T''}\left(\frac{\sigma_1}{T}-\frac{\sigma_2}{T}\right)dT \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The values of this integral were obtained by means of drawing a σ/T, T-diagram (Fig. 11), based on the σ-values obtained, and a graphical integration of the areas, denoted as A, B and C. A comparison between these values of the integral and the corresponding differences in thermo-electric force obtained in Table V gives the results in Table VI.

The deviations from equation (4) are entirely within the limits of the accidental errors; hence this control gives no reason to assume the presence of any methodical or systematic errors in the measurements of the THOMSON effect.

§ 5. The measurements give no sufficient evidence as to the manner in which the THOMSON effect varies with temperature in the proximity of the absolute zero point, neither concerning the question whether σ will there approach to 0 [1]). Also because of the theory given by SOMMERFELD [2]), we think measurements at the temperatures, obtainable with liquid helium, very desirable.

If, according to NERNST [3]) $\lim e = 0$ for $T \to 0$, comparatively great values for $\frac{\sigma_1-\sigma_2}{T}$ would be expected for the thermocouple silveralloy-copperalloy at temperatures lower than those used in these measurements. From $e = 4.74 \times 10^{-6}$ at $T = 17.3$ (see Table IV) follows according to equation (4) for the mean value between $T = 0$ and $T = 17.3$: $\frac{\sigma_1}{T}-\frac{\sigma_2}{T}=\frac{4.74\times10^{-6}}{17.3}=27\times10^{-8}$ volt/degree^2, whereas the greatest

[1]) W. H. KEESOM. These Proc. **16**, 1913, p. 236. Comm. Leiden Suppl. N°. 30*b*.
[2]) A. SOMMERFELD. ZS. f. phys. **47**, 43, 1928.
[3]) W. NERNST. Theoretische Chemie 1913, p. 753.

value of this difference within the range of these measurements amounts to 5.4×10^{-8} volt/degree^2 (at 50° K.). The fact, that even at the liquid-

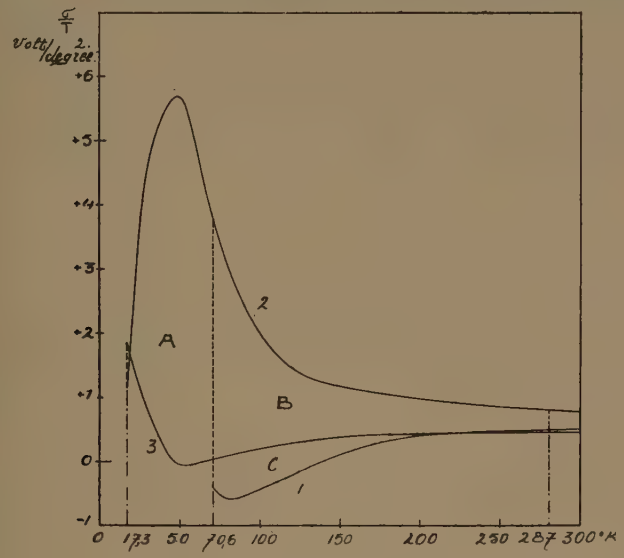

Fig. 11.

1. copper. 2. copperalloy. 3. silveralloy.

TABLE VI.

Differences in Thermo-electric Force per Degree calculated from THOMSON-effect and direct observed				
Microvolt per degree				
°K		$e'' - e' = \int_{T'}^{T''} \frac{\sigma_1 - \sigma_2}{T} dT$	$e'' - e'$ observ.	Diff.
T'	T''			
Silveralloy against Copperalloy				
17.3	70.6	— 2.24	— 2.18	— 0.06
70.6	280.7	— 2.01	— 2.19	+ 0.18
Copper against Copperalloy				
70.6	280.7	— 2.53	— 2.62	+ 0.09

helium temperatures still comparatively great values for σ/T appear, follows also from the measurements concerning thermo-electric forces at that temperature by KAMERLINGH ONNES and HOLST [1]).

We are glad to render our thanks to the NOBEL-committee for physics, which supported this research by a subsidy.

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1) H. KAMERLINGH ONNES and G. HOLST. These Proceedings **17**, 1914, p. 760. Comm. Leiden N°. 142c.

Physics. — *The melting curve of hydrogen to* 245 kg/cm². By W. VAN GULIK † and W. H. KEESOM. (Comm. N⁰. 192*b* from the Physical Laboratory at Leiden.)

(Communicated at the meeting of November 24, 1928).

§ 1. This communication refers to a first continuation of the investigation on the melting curve of hydrogen, the first results of which have been published in Comm. N⁰ 184*a* [1]). Whereas then the measurements went from the triplepoint to a pressure of 55 kg/cm², the measurements are continued now to a pressure of 245 kg/cm², corresponding to a temperature of 20.35° K.

§ 2. The method was the same as that of Comm. N⁰. 184*a*. The apparatus described in Comm. N⁰. 184*b* [2]) was used, except that the glass tube *F* was omitted, and the cryostat, into which liquid hydrogen was now poured, was simpler.

The temperature was measured with the platinum thermometers Pt_{32} and Pt_{36}, which had been used too for the measurements of Comm. N⁰. 184*a*, and which were calibrated with a helium thermometer. The pressure was measured with a metal manometer going to 400 kg/cm² [3]). This manometer was, for the purpose of a better observation, provided for this experiment with a more accurately divided scale. The intention was to calibrate this manometer again after the experiments with the pressure-balance of the VAN DER WAALS-foundation at Amsterdam; this intention was frustated however, because the manometer became defective before this could be done. The corrections were deduced therefore from a comparison with the above mentioned pressure-balance to 253 kg/cm², made in Sept. 1926. The change of the zeropoint, which was caused by the taking off and replacing of the pointer on the occasion of the affixing of the new scale, was deduced from a comparison with the closed hydrogen-manometer M_{120}, performed on Dec. 17th 1926 and Jan. 25th 1927.

§ 3. The results are given in Table I.

In Fig. 1 these results are represented, indicated by ⊙, together with those of Comm. N⁰. 184*a*, indicated by △, It appears that the results of

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[1]) H, KAMERLINGH ONNES and W. VAN GULIK. These Proceedings **29**, 1184, 1926.

[2]) W. H. KEESOM. These Proceedings **29**, 1136, 1926.

[3]) The same as mentioned in Comm. Leiden N⁰. 184*b* p. 14 note 2.

these new measurements can be very well combined with the previous ones to a smooth curve.

TABLE I.

Melting curve of hydrogen (Jan. 18, 1927)		
Temp. °K	Pressure	
	Kg/cm²	Atm.
20.34⁵	245	237
19.69	215	208
18.36	161	155
17.27	114	110
16.31	97	94

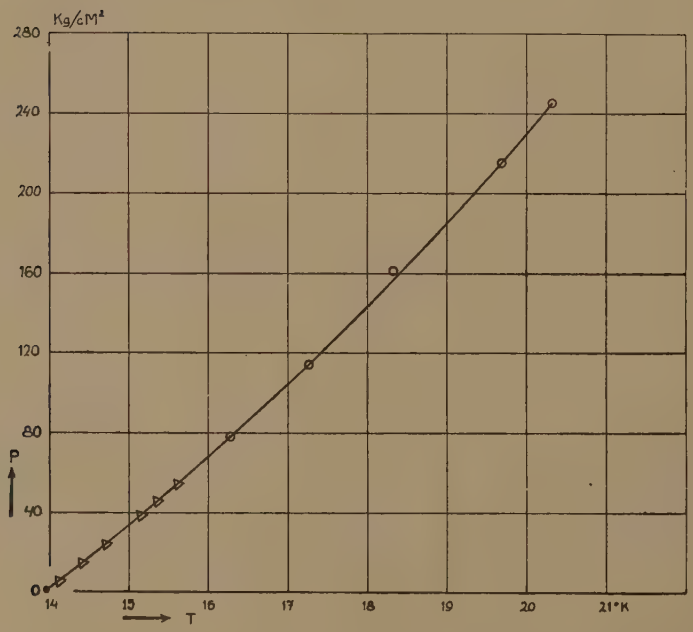

Fig. 1.

The upwards curvature, which the melting curve already proved to possess in the part then investigated (compare Fig. 1, Comm. N⁰. 184*a*), has now been further accentuated.

Geology. — *On Tertiary Foraminifera from Curaçoa.* By L. RUTTEN.

(Communicated at the meeting of November 24, 1928)

The material to be described in this paper, was collected in the year 1921 by the engineer G. MOLENGRAAFF in the island of Curaçoa, and was kindly subjected to our examination. It was taken from the Seru di Cueba, a hill 90 m high in the north of the island [1]). According to MOLENGRAAFF's information the nucleus of this hill consists of diabase, overlain by almost horizontal tertiary limestones, whose thickness — judging from a profile in my possession — may amount to about 40 m. On the north side of the Seru di Cueba these limestones are overlain by quaternary limestones, separated locally by a thin basal conglomerate from the Tertiary. The quaternary limestones incline very slowly seaward. The rocks to be described here have been taken from the southern slope of the Seru di Cueba ; a number of detached fossils are derived from the south-, and the east slope ; some boulders from the quaternary conglomerate were collected on the north side of the hill.

Some months ago an essay was published by R. KOCH [2]), who also occupies himself with tertiary foraminifera from Curaçoa. In this essay the author describes material collected by Dr. STAUFFER and MOLENGRAAFF on the "Seru Kenepa". According to information obtained from Mr. MOLENGRAAFF there is a mistake here : the Seru Kenepa, situated 3 km to the south of Seru di Cueba consists entirely of diabase, while KOCH's material has been taken from the Seru di Cueba, just as mine has, but it has been collected on the western slope of the hill. There would be very little inducement to publish the result of the examination of MOLENGRAAFF's material, if not considerable differences had appeared to exist between the limestones, examined by KOCH and those studied by me. These differences imply that various levels of the Old Tertiary seem to occur in the Seru di Cueba.

The material studied by KOCH contained besides entirely indifferent foraminifera and lithothamnia numerous Operculina and Alveolina, an occasional, very small nummulite, a few fragments of Orthophragmina (Discocyclina) and many Lepidocyclines. It is especially the latter that were examined more closely by the writer and that helped him in determining the

[1]) Topographical map of Curaçao. 1 : 20000. Sheet III.

[2]) R. KOCH, Tertiärer Foraminiferenkalk von der Insel Curaçao, Ned. West Indien Eclogae geologicae Helvetiae. **21**, 1928, p. 51—56. T. III.

age of the sediments. We shall have to take account of the results. KOCH has succeeded in determining the following species :

Polylepidina sp.	common.
Pliolepidina panamensis Cushm.	very common.
Isolepidina cf. Raulini Lem. et Douv.	rare.
Isolepidina Macdonaldi Cushm.	very common.
Isolepidina trinitatis Douv.	common.
Isolepidina pustulosa Douv.	common.
Isolepidina cf. Hubbardi Hodson.	rare.
Nephrolepidina Tournoueri Lem. et Douv.	rare.
Nephrolepidina Morgani Lem. et Douv.	rare.
Nephrolepidina sumatrensis Brady	common.
Nephrolepidina yurnagunensis Cushm.	common.
Lepidocyclina (subgen. ind.) curasavica n.s.	the most common form.
Lepidocyclina sp. sp.	

It stands to reason that Lep. curasavica cannot be used as an index for age determination. I also wish to leave out of consideration the two species, whose determination is not quite definite, viz. Isolepidina cf. Raulini and Isolepidina cf. Hubbardi, which moreover are of rare occurrence. As to the first, there must be a mistake, for Lepidocyclina Raulini belongs to Eulepidina, not to Isolepidina. Of the remaining species we may say what follows :

Polylepidina sp. So far as I know all the Polylepidinae described to this day originate from the Eocene [1]).

Pliolepidina panamensis Cushman. T. WAYLAND VAUGHAN [1]) reports of this species that it "is probably eocene, may be oligocene" (l.c. pl. 33, fig. 1).

Isolepidina Macdonaldi Cushman is according to VAUGHAN (l.c.) typical of the Eocene of Panama.

Isolepidina Trinitatis H. Douv. Originally this species is referred to Oligocene. But in the latter years it is regarded as being of Eocene age by H. DOUVILLÉ [2]) as well as by VAUGHAN (l.c.).

Isolepidina pustulose H. Douv. Referred with some reserve by H. DOUVILLÉ (l.c.) to the Eocene ; by VAUGHAN without reserve (l.c.).

Nephrolepidina Tournoueri Lem. et Douv. known from the Oligocene of E. Mexico (VAUGHAN l.c.) and from the Oligomiocene of Trinidad (H. DOUVILLÉ l.c.).

Nephrolepidina Morgani Lem. et Douv. known from the Oligocene of Cuba and Mexico (VAUGHAN l.c.).

Nephrolepidina sumatrensis Brady. According to CUSHMAN [3]) this form belongs to the Oligocene of Cuba. However, the pictures he gives are by no means typical. They do not show at all that the form he reproduces are really N. Sumatrensis. With this reserve we must also accept KOCH's specific determination.

Nephrolepidina yurnagunensis Cushman is mentioned from the Oligocene of Cuba.

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[1]) T. WAYLAND VAUGHAN, American and European tertiary larger Foraminifera. Proc. Palaeont. Society, reprinted from the Bull. Geol. Soc. America. **35**, 1924, p. 785—822. pls. 30—36.

[2]) H. DOUVILLÉ, Mém. Soc. géol. de France, Nouvelle série I, 2, 1924.

[3]) J. CUSHMAN, Un. States Geol. Survey. Professional paper 125 D, 1920.

So, when all comes to all, in the rocks of KOCH a number of Polylepidinae and Isolepidinae that point to Eocene, occur together with Nephrolepidinae that suggest Oligocene. From this KOCH concludes :

"Das Alter des Foraminiferenkalkes..... ist durch das Auftreten mehrerer typischer Oligocänlepidocyclinen als Oligocän bestimmt."

This conclusion looks decisive, but it is liable to objection, since besides "typische Oligocänlepidocyclinen" also "typische Eocänlepidocyclinen" occur.

Now, the rocks examined by me have quite a different fauna, which will be seen first of all from a short description.

G. 574. 1928. D. 10606 [1]) is a rock very rich in Lithothamnium, coarsely porous, without clastic material. The Lithothamnia are very fine-meshed. Unmistakable remains of very small megalospherical Nummulites occur, of which three specimens have the following measures :

N°. 1. diam. 1 mm thickness 0.4 mm embryonal chamber interior 0.175 mm ? 3 whorls.
N°. 2. diam. 1½ mm thickness 0.8 mm embryonal chamber interior 0.16 mm ? 5 whorls.
N°. 3. diam. 1 mm thickness 0.8 mm embryonal chamber interior 0.1 mm 3 whorls.

These three specimens agree well inter se ; but the material is not sufficient to make a specific determination. The only species known with which the fossils may possibly be compared is Numm. variolarius Sow. from the Upper Eocene.

G. 575. 1928. (large, unnumbered section). Light ochre limestone with numberless inflated lepidocyclines with a diameter of from 3—5 mm, partly megalospherical. Under the microscope we see besides very numerous, extremely fine-meshed Lithothamnia, a rare small nummulite and sparse echinid-rests, and a large number of lepidocyclines. The largest are 8 mm in diameter, and are microspherical. Most of them are megalospherical and have a diameter of from 2½—5 mm. All the well-preserved specimens are strongly inflated and have distinct columns, which, however, are few and far between. Where they are visible the initial chambers are isolepidine, with a diameter of about 0.5 mm (fig. 1—5). All these features fairly agree with those of Isolepidina Trinitatis H. Douv.

G. 576. 1928. D. 10607, 10618—10622. Rounded fragment of an ochre limestone with numerous close-meshed Lithothamnia, with Lepidocyclines and Operculina and with sparse Textularidae and Rotalidae, a rare small Nummulite and possibly a Carpenteria. The Lepidocyclines belong to various species. May be there occur a few specimens of Isolepidina Trinitatis. The bulk of the fossils may be classed with two species. In the first place we find flat, small Lepidocyclines with a diameter of less than 3½ mm and a thickness of less than 0.6 mm. There occur sparse, irregular papillae. The number of layers of lateral chambers is mostly 3—4 in the middle, sometimes, but rarely 5 or 6 ; towards the borders this number dwindles down to 0. The embryonal apparatus seems to be variable ; it may have a large diameter. I have never been so fortunate as to obtain a satisfactory horizontal section of the embryonal apparatus in my preparations, so that I do not know to which subgenus of Lepidocyclina the fossils belong. However. they generally agree fairly well with Lep. (Polylepidina) proteiformis Vaughan from the Eocene of Mexico (fig. 10—16 and Plate Fig. E).

The second species is very small and considerably inflated, the horizontal diameter is only 2—3 mm. We did not find a beautiful horizontal-median section, but the embryonal

[1]) The number with the date refer to the annual catalogue of the Geol. Inst. of Utrecht, the number with "D" refer to the collection of slides.

apparatus is decidedly pliolepidine (fig. 26—33, Pl. fig. C). All the characteristics agree well with Pliolepidina Tobleri H. Douv. from the Upper Eocene of Trinidad.

G. 577. 1928. D. 10608. Rounded white Lepidocyclina-Lithothamnium-limestone, possibly

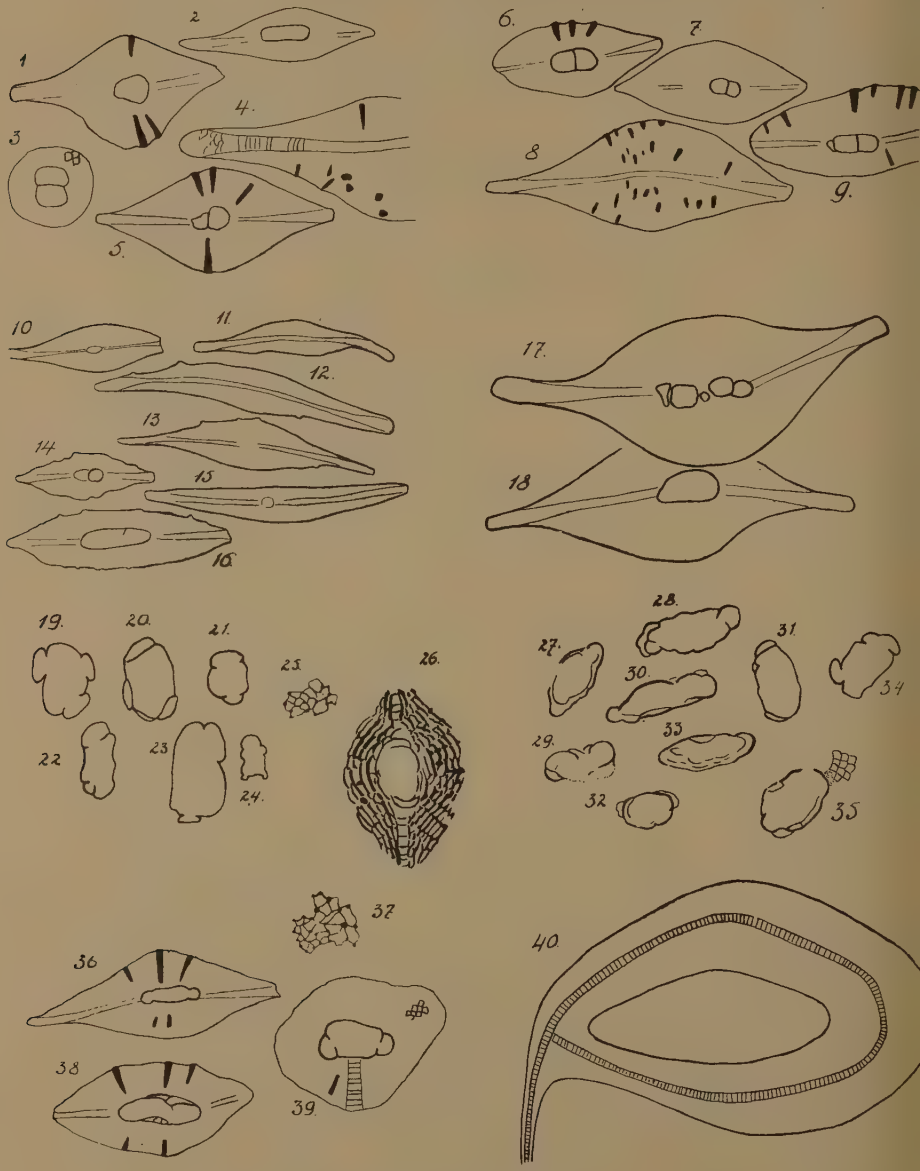


Fig. 1—40

with Nummulites. At least three species of Lepidocyclines occur: 1. Pliolepidina Tobleri H. Douv. (fig. 35); 2. a very flat species; 3. a columnless or nearly columnless, highly inflated species with a diameter of less than 5½ mm, a maximal thickness of 1.6 mm. with 10—12 layers of lateral chambers, and with an irregular embryonal apparatus (fig. 17, 18). I cannot give a specific determination of this species.

G. 578, 1928, D. 10609, 10623, 10624. Rounded fragment of a porous, yellowish white lepidocyclina-lithothamnium-limestone with sparse nummulites and textularids. So far as can be ascertained all the Lepidocyclines belong to Pliolepidina. Some of them very much resemble the Pliolepidina of the preceding rocks (fig. 34). Others are somewhat larger (diameter 2—3, sometimes 5 mm, thickness 1—1.2 mm); they possess more distinct columns, and have 7—10 layers of lateral chambers (fig. 36—39). These differences, however, are not large enough, to warrant a complete separation from Pliolepidina Tobleri. Probably we have to do with a varietal form of Pliolepidina Tobleri.

G. 579. 1928. D. 10610. A yellow-specked, porous limestone with ill-preserved Lepidocyclines, Lithothamnia and Operculina. Different species of Lepidocyclina occur in the rock, of which I could determine only Pliolepidina Tobleri with certainty.

G. 580. 1928. D. 10611—10617. 10625, 10626. White crumbling limestone with Lepidocyclinae, Lithothamnia and Operculina. Different species of Lepidocyclina occur daceae. A number of small Lepidocyclinae could be isolated. Six of these had a very irregular embryonal apparatus, which was invariably pliolepidine (fig. 19—24, Plate fig. D). The diameter of the megalospherical forms is 3—4.5 mm. Two microspherical forms that apparently belonged to them, had a diameter of 4—6 mm. It is especially with these that we observe distinctly very tiny columns, such as are typical in Pliolepidina Tobleri (fig. 25). The median chambers also resemble those of Pl. Tobleri, but they are somewhat smaller (radial 0.055—0.060 mm tangential 0.07—0.08 mm). The forms may certainly be referred to Pliolepidina Tobleri. Besides this far prevailing form small flat Lepidocyclines occur, probably belonging to the same species as those of no. 576 (?Polylepidina proteiformis).

G. 581, 1928. (Large not numbered slide). Very porous Lithothamnium-Lepidocyclina-limestone with at the least five different species of Lepidocyclina:

1. An inflated form of 3 by 1½ mm with rather large columns (Isolepidina Trinitatis).
2. Pliolepidina Tobleri.
3. Very flat, small lepidocyclines of 3½—5 by 0.75 mm with from 5 to 7 layers of lateral chambers, probably the same form as in 576 (?Polylepidina proteiformis).
4. A semi-inflated, column-poor or column-free form of 5 by 1½ mm, perhaps the same as in 577.
5. A microspherical form of 10 mm diameter, which I cannot determine farther.

G. 582. 1928. (Large, unnumbered slide). A Lithothamnium-Lepidocyclina-limestone, much like no. 581, but which also contains a few small nummulites. The Lepidocyclina-fauna is the same as in 581 (figs. 6—9).

G. 583. 1928. D. 10627—10636. Separate nummulites, to be classed as a new species Numm. striatoreticulatus (figs. 41—50, Plate figs. F—J).

G. 584. 1928. D. 10637, 10638. A few separate specimens of a very remarkable Lepidocyclina, which belongs to a new species L. brachiofera (fig. 40, Plate A, B). In the rock that is still attached to the fossils, also Nummulites, and Operculina, besides a small Lepidocyclina occur.

G. 585. 1928. 10639. A few loose specimina of a very indifferent Operculina, which I did not endeavour to determine specifically.

G. 586. 1928. D. 10640—10641. Limestone with numerous specimens of Isolepidina Trinitatis H. Douv.

G. 587. 1928. D. 10642—10648. Lepidocyclina-Lithothamnium-limestone with Operculina. The Lepidocyclines belong partly to the very flat form that is also found in other rocks. A characteristic of this rock is, however, the occurrence of large, microspherical,

medially highly inflated, columned Lepidocyclina, whose diameter may amount to 16 mm. It may probably be referred to Lepidocyclina curasavica Koch.

Lastly there are a few more separate Lepidocyclinae in the collection, which, however, I did not venture to determine specifically, for want of sufficient microscopical sections.

Besides the samples of tertiary limestone there was at my disposal one fragment of quaternary limestone (*G. 573, 1928*), which differs from the tertiary rocks in every respect. It contains many grains of older rocks, viz. of radiolarite and of totally weathered diabase; there occur in it colonies of Lithothamnia, that are far widermeshed than the tertiary; the tertiary foraminifera are lacking and on the other hand rather many Amphistegina are present.

It appears now that in the tertiary rocks the following typical foraminifera could be identified:

Pliolepidina Tobleri H. Douv.	very numerous.
Pliolepidina Tobleri H. Douv. var.	sparse.
Isolepidina Trinitatis H. Douv.	rather numerous.
Lepidocyclina brachiofera nov. sp.	sparse.
Lepidocyclina sp. (?Polylepidina proteiformis Vaughan)	very common.
Lepidocyclina curasavica Koch.	not common.
Lepidocyclina sp. ind. div.	
Nummulites striatoreticulatus nov. sp.	not common.

When comparing these faunula with the one described by KOCH we see at once essential differences. Nephrolepidines, of which in KOCH's material two species were numerous, are absolutely lacking in the rocks examined by me: in the very numerous preparations I did not detect any. Lepidocyclina curasavica, the most common form with KOCH, occurred only once with me in a single sample; even there the determination is not absolutely certain. Isolepidina Macdonaldi, "very common" in KOCH's material, was not found in my rocks, still it could hardly have been overlooked; the same applies to Pliolepidina panamensis. On the contrary KOCH has not found Pliolepidina Tobleri, the most common form in my material, and in his material the typical Nummulites are absent.

Whereas KOCH may say with some justice that the rocks examined by him are of oligocene age, on the basis of my material of fossils I feel justified in saying with equal justice that *the rocks are eocene.*

The new and remarkable nummulite, occurring in the material examined, is the first typical nummulite that, so far as I know, has been described from America. In VAUGHAN's résumé we read that of nummulites from America only N. parvula Cushman from the Eocene of St. Bartholomew has been pictured and described; this form, however, is a degenerated nummulite, which moreover is insufficiently known from a single section. DOUVILLÉ has reported [1]) different nummulites, but some of them are Operculines (N. Willcoxi, N. Heilprini, N. Floridensis), others again are not so

[1]) H. DOUVILLÉ, C.R. Acad. Sc. Paris **161**, 1915, p. 87—93. **164**, 1917, p. 841—847

well described (N. vascus, N. aff. striatus, N. irregularis) that we can certify their determination, while pictures are entirely lacking.

Palaeontology.

In the foregoing two new species have been mentioned that still require a description, while something must also be said about one of the known Lepidocyclines.

Lepidocyclina (Pliolepidina) *Tobleri H. Douv.* (figs 19—35, Pl. figs C. D.). Among the rocks of Curaçoa this is one of the commonest if not the most common of all species. As late as 1924 DOUVILLÉ (l.c.) still believes in the possibility that Pliolepidina Tobleri is a "teratologic form" of Isolepidina Trinitatis. Nowadays we are bound to look upon Pliolepidina as a proper subgenus of Lepidocyclina. It is not only that in America besides P. Tobleri other species are known (P. duplicata Cushm. and P. panamensis Cushm.) but Pliolepidinae have also been found in the Dutch Indies [1]). In accordance with all this the fossils of Curaçoa can be very well discriminated from Isolepidina Trinitatis, even when eliminating the difference in the embryonal apparatus. The structure of P. Tobleri is smaller, looser and its skeletal columns are much more delicate than those of I. Trinitatis. DOUVILLÉ's view that P. Tobleri should be a teratologic form of I. Trinitatis, cannot be held any longer.

Lepidocyclina brachiofera nova species (fig. 40 and Plate figs A, B). Of this extremely wonderful form only three specimens were present, of which only one is fullgrown (fig. A). The smallest has been used for a controlling horizontal section. The largest specimen has a diameter of rather more than 3 cm ; the middle specimen of 2 cm. The full-grown individual has a median tubercle not sharply outlined ; the upper surface of the plasmostracum is covered with very fine columns, not continuing far interiorly. The most characteristic feature is, that in its early existence the plasmostracum displays at the periphery undulations which in the fullgrown individual are transformed into hollow arms, of unknown length, but whose basal part is still to be observed in fig. A as well as fig B (a transverse section of one arm is shown in fig. 40). In the full-grown specimen not less than ten of such arms have existed. When inspecting the transverse section of an arm it appears that in its walls all the layers of the plasmostracum may be distinguished. In the middle there is a central cavity, then follows a circulating band of layers of lateral chambers, which are the continuation of the lateral-chamber layers at the underside of the test, farther outwardly follows a circulating band of median chambers (hatched in fig. 40), still farther towards the outside a second band of layers of lateral chambers follows which represents the continuation of the lateral-chamber layers at

[1]) I. VAN DER VLERK, Het genus Lepidocyclina in het indopacifische gebied. Dienst van den Mijnbouw. Wetensch. Meded. 8. 1928.

the upper side of the test. But the structure is still somewhat more complicate. The "lowermost" layers of lateral chambers simply bend round in

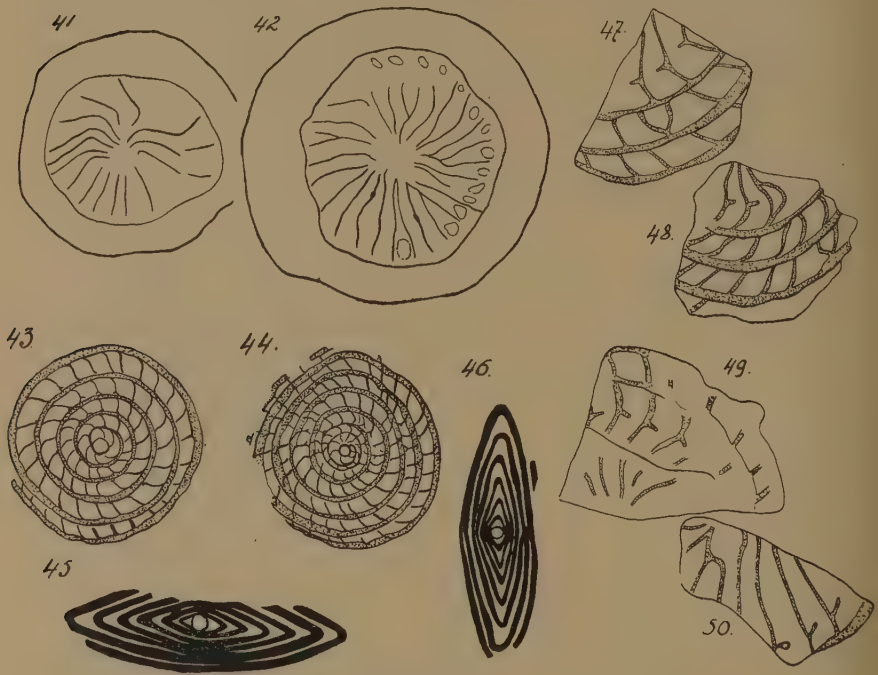


Fig. 41—50.

the arm but the median-chamberlayer, and the uppermost layers of lateral chambers extend still further past the "suture" of the arm, so that the result is that the arm is provided with a longitudinal leaf-shaped appendix (fig. 40). It is a great pity, that only few individuals of this wonderful species have been found, as it would certainly have been worth our while to make a reconstruction of one whole individual.

Nummulites striatoreticulatus nov. sp. (figs 41—50, Pl. figs F, G, H, I, J). These fossils, which occur in a rather large number (between 50 and 100 specimina) in the material of G. MOLENGRAAFF, cannot be classed with any known nummulite. Not only do they differ in the ordinary features (diameter and height, number of whorls, number of septa per whorl) from all nummulites, known to me, in so far as in N. striatoreticulatus such a combination of these features occurs as is not known to exist in any other species, but moreover the structure of the septa exhibits peculiarities that I have not found mentioned anywhere. All the fossils but one are megalospherical. The only microspherical specimen is rather large, c.a. 10 by

2.5 mm. It possesses 10 annuli ; in the ninth whorl about 38 septa occur. For some megalospherical specimens we subjoin the numerical data.

	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7
diameter (mm)	4	4.7	3.8	4.7	4.7	4.5	4.
thickness (mm)	1.3	1.7	——	——	——	——	——
initial chamber diameter (mm)	0.2	0.3	0.32	0.27	0.25	0.3	0.23
number of whorls	6½	6½	5	6½	6¼	5½	5½
Septa in 5th or 6th whorl	——	——	22	28	25	27	24

It will be seen from this table that the various individuals agree fairly well inter se. It should be observed, however, that the sections were taken from the larger specimens, and that besides them many small individuals (youth-forms?) occur. The fossils are characterized by small thickness (figs 45, 46), closely serried whorls, and very long chambers. The septa arise on the inside of the outer septum, at a slight angle, and while gradually curving, they reach, at a rather large angle, the inner septum (Pl. figs G—I, figs 43, 44).

When noticing, on the surface of the nummulites (Pl. fig. F) or after grinding off a tangential-horizontal surface, the septa (figs 41, 42), they always appear to run radially with slight, irregular bends ; this identifies the nummulites with the striated nummulites. Of real granulae absolutely nothing is to be seen ; occasionally I saw some faint, thickenings on the septa (fig. 42). But in the usual striated structure there appears a complication, unknown to me in any European or Asiatic nummulite (figs 43, 44, 47—50. Plate figs G—I). Namely while in the median plane the septa are simple, a little beyond the median plane they exhibit an anteriorly directed process (slightly schematically in figs 43, 44 ; very beautifully in Pl. fig. H, top of the picture). I succeeded in opening one specimen of a fossil, so that I could study the interior of some whorls. Fig. 50 shows the inside of one whorl ; in figs 47—49 we are face to face with three disk-shaped, fragments of annuli fitting to each other. In all these cases we see distinctly the anteriorly directed processes at a number of septa. Now, in the case of a sub-median cross-section it may happen that on co-operation of the effects of different annuli it would seem as if a reticulate structure appears in part of the section, which in fact does appear in all likelihood, as the above-described processes may extend as far as an anteriorly directed septum.

This can be seen very distinctly in the middle of Pl. fig. G and also, but less distinctly, in fig. H. It should be pointed out that this retiform or pseudo-retiform structure has a quite different origin from that of the netlike structure known in the true reticulate nummulites from Europe and Asia.

Further research will have to show if we have to do here with a new group of nummulites, which may have been confined to America.

EXPLANATION OF THE FIGURES.

Figs 1—5. Lepidocyclina (Isolepidina) Trinitatis H. Douv. (in fig. 4 a microspherical form). From G. 575, 1928. × 13.

,, 6—9. Lepidocyclina (Isolepidina) Trinitatis H. Douv. (Fig. 8 microspherical). From G. 582, 1928. × 13.

,, 10—16. Lepidocyclina aff. ??Polylepidina proteiformis Vaughan. From G. 576, 1928. 10, 11, 14 from D. 10618, 12 from D. 10607, 13 from D. 10622, 15, 16 from D. 10619. × 13.

,, 17. 18. Lepidocyclina sp. From G. 577, 1928. D. 10608. × 13.

,, 19—25. Pliolepidina Tobleri H. Douv. From G. 580, 1928. 19 from D. 10611, 20 from D. 10612, 21 from D. 10613, 22 from D. 10614, 23 from D. 10615. 24 from D. 10625, 25 from D. 10617. × 13.

,, 26. Pliolepidina Tobleri H. Douv. From G 576, 1928, D. 10621. × 16.

,, 27—33. Pliolepidina Tobleri H. Douv. From G. 576, 1928. 27 from D. 10607, 28, 31, 33 from D. 10620, 29 from D. 10607, 30 from D. 10619, 32 from D. 10621. × 13.

,, 34. Pliolepidina Tobleri H. Douv. From G. 578, 1928, D. 10623. × 13.

,, 35. Pliolepidina Tobleri H. Douv. From G. 577, 1928, D. 10608. × 13.

,, 36—39. Pliolepidina Tobleri H. Douv. var. From G. 578, 1928, 36, 37 from D. 10809, 38, 39 from D. 10623.

,, 40. Lepidocyclina brachiofera nov. sp. Transverse section of one of the "arms". G. 583, 1928. × 8.

,, 41, 42. Nummulites striatoreticulatus nov. sp. Seen from above and ground tangentially. × 10.

,, 43. Nummulites striatoreticulatus nov. sp. Horiz. section. D. 10636. × 8.

,, 44. Nummulites striatoreticulatus nov. sp. Horiz. section. D. 10634. × 6.

,, 45. Nummulites striatoreticulatus nov. sp. Vert. section. D. 10628. × 9.

,, 46. Nummulites striatoreticulatus nov. sp. Vert. section. D. 10629. × 9.

,, 47—50. Nummulites striatoreticulatus nov. sp. Interior aspect of an opened plasmostracum. × 11.

Plate figs A, B. Lepidocyclina brachiofera nov. sp. About natural size.

,, ,, C. Pliolepidina Tobleri H. Douv. From G. 576, 1928. D. 10619. Same as in fig. 30. × 22.

,, ,, D. Pliolepidina Tobleri H. Douv. From G. 580, 1928. D. 10625. Same as in fig. 24. × 15.

,, ,, E. Lepidocyclina aff. ??Polylepidina proteiformis Vaughan. From G. 576, 1928. D. 10619. Same as in fig. 16. × 22.

,, ,, F. Nummulites striatoreticulatus nov. sp. × 10.

,, ,, G. Nummulites striatoreticulatus nov. sp. × 10. D. 10631.

,, ,, H. Nummulites striatoreticulatus nov. sp. × 10. D. 10630.

,, ,, I. Nummulites striatoreticulatus nov. sp. × 10. D. 10635. Same as in fig. 42.

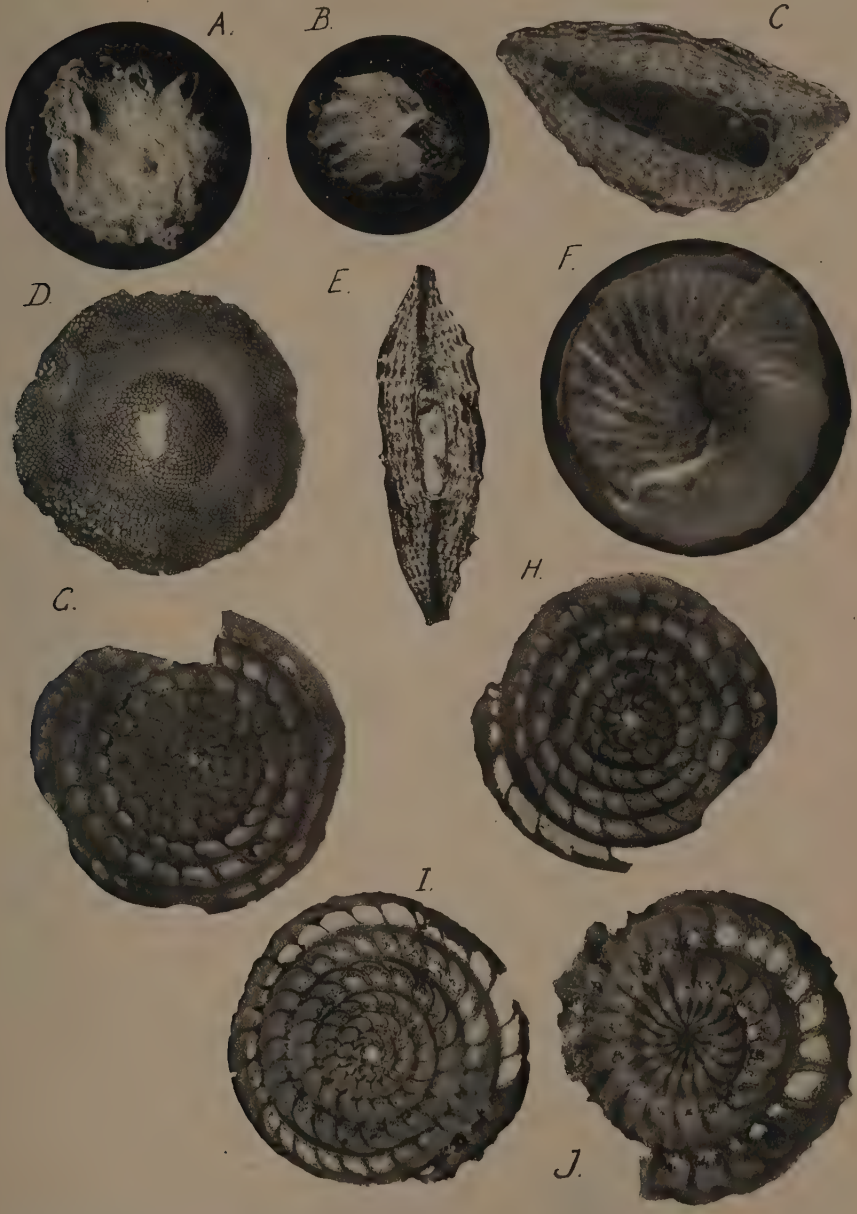
,, ,, J. Nummulites striatoreticulatus nov. sp. × 15. D. 10633.

All the magnifications are approximative.

The drawings are made either with camera lucida (Nos. 1—42, 47—50) or drawn over a photo, after which the photographic print was washed away.

L. RUTTEN: „TERTIARY FORAMINIFERA FROM CURAÇOA".

PLATE Fig. A—J



Mathematics. — *Zwei Kongruenzen von rationalen Raumkurven vierter Ordnung.* By Prof. JAN DE VRIES.

(Communicated at the meeting of December 22, 1928).

1. Vorgegeben seien ein Büschel (R^3) von kubischen Regelflächen mit Doppelgerade d und ein Büschel (R^2) von quadratischen Regelflächen durch d. Die Basiskurve σ^5 von (R^3) hat d zur Quadrisekante; die Basiskurve σ^3 von (R^2) trifft d zweimal.

Die beiden Büschel erzeugen eine *Kongruenz* $[\varrho^4]$ von *rationalen* Kurven ϱ^4, welche σ^5 je in 6 Punkten, σ^3 je in 5 Punkten treffen.

2. Es sei δ eine durch d gelegte Ebene; (R^3) und (R^2) bestimmen in δ zwei Stralenbüschel um die Spuren S_5 und S_3 der Kurven σ^5 und σ^3.

Als *Bild* einer ϱ^4 betrachte ich den Punkt R, den ϱ^4, ausserhalb d, mit δ gemein hat.

Der *singuläre* Punkt S_3 ist das Bild sämmtlicher ϱ^4, welche (R^2) auf der durch S_3 gelegten Regelfläche R^3 bestimmt. Analog ist der *singuläre* Punkt S_5 das Bild aller ϱ^4, welche auf der durch S_5 bestimmten R^2 liegen.

Die Gerade $S_3 S_5$ bildet eine Figur ab, welche aus ihr und einer sie schneidenden kubischen Kurve ϱ^3 besteht.

Offenbar gibt es ∞^1 Figuren (ϱ^3, r). Die Geraden r bilden eine *Regelfläche achten Grades*, $(r)^8$, mit siebenfacher Leitgeraden d und Leitlinien σ^3, σ^5.

3. Betrachten wir das System Λ der ϱ^4, welche eine vorgegebene Gerade l treffen. Die Bildkurve λ des Systems hat einen Doppelpunkt in S_5, einen dreifachen Punkt in S_3; sie ist demnach eine $\lambda^5 (S_3^3, S_5^2)$. Da sie *rational* ist, besitzt sie noch zwei Doppelpunkte; diese bilden je eine ϱ^4 ab, welche l *zweimal* trifft. Ihre Stützpunkte liegen in den beiden gemeinschaftlichen Paaren der Involutionen I^2 und I^3, welche (R^2) und (R^3) auf l einschneiden.

Zwei Bildkurven λ^5 haben $5^2 - 3^2 - 2^2$, also 12, Punkte R gemein; das System Λ liegt somit auf einer *Fläche* 12$^{\text{en}}$ *Grades*, Λ^{12}, mit Doppelkurve σ^5, dreifacher Kurve σ^3 und zwei Doppelkurven ϱ^4. Da ihr Schnitt mit δ aus λ^5 und d besteht, ist d eine *siebenfache* Gerade.

4. Die Kurven ϱ^4, welche d in einem Punkte D treffen, bilden somit eine *Fläche* Δ^7.

Jede Ebene durch d enthält zwei sich in D treffende Geraden r_2, r_3,

welche den sich in D berührenden Flächen R^2, R^3 angehören. Hieraus erhellt dass jeder R^2 eine R^3, jeder R^3 hingegen zwei R^2 sind zugeordnet. Die Fläche Δ^7 ist das Erzeugniss dieser Verwandtschaft; demnach ist σ^3 Doppelkurve, σ^5 einfache Kurve auf Δ^7. Die Bildkurve der auf Δ^7 liegenden ϱ^4 ist eine $\delta^3 (S_3^2, S_5)$; also ist d eine *vierfache* Gerade.

Zwei Kurven δ^3 haben 4 Punkte R gemein; durch je zwei Punkte D der Geraden d gehen somit vier Kurven ϱ^4.

In einer Ebene Φ durch D erzeugen die durch (R^2) und (R^3) bestimmten Büschel (c^2) und (c^3) eine Kurve Φ^7, mit 3 Doppelpunkten auf σ^3. Diese Kurve hat mit einer c^2 3 Punkte einer ϱ^4 und 6 Punkte auf σ^3, somit 5 Punkte in D gemein.

Demnach ist D ein *fünffacher* Punkt der Fläche Δ^7.

5. Die Flächen Δ^{12} und $(r)^8$ haben, ausser den vielfachen Linien $d(7^2)$, $\sigma^3(3\times 3)$, $\sigma^5(2\times 5)$, 28 Geraden r gemein, von denen 8 die Gerade l treffen (§ 2). Den übrigen 20 Geraden entsprechen 20 Kurven ϱ^3, welche sich auf l stützen; demnach bilden die Kurven ϱ^3 eine *Fläche* Σ^{20}.

Die Flächen R_0^2 und R_0^3, welche durch $r_0 (S_3 S_5)$ gehen, erzeugen noch eine ϱ_0^3, welche r_0 in einem Punkte R_0 trifft. Die Fläche R_0^2 schneidet σ^5, ausserhalb d, in 6 Punkten R, enthält somit ausser (r_0, ϱ_0^3) noch 5 Figuren (r, ϱ^3); diese werden in S_5 abgebildet. Daher ist S_5 ein 5-facher Punkt der Bildkurve σ des Systems Σ. Analog ergibt sich, dass S_3 ein 4-facher Punkt ist. Beachtet man, dass $S_3 S_5$ noch den Punkt R_0 enthält, so erhellt, dass jene Bildkurve eine $\sigma^{10} (S_3^4, S_5^5)$ ist.

Die Kurven $\sigma^{10} (S_3^4, S_5^5)$ und $\lambda^5 (S_3^3, S_5^2)$ bestimmen 28 Punkte R; ihnen entsprechen $8\,\varrho^3$, welche l nicht treffen; es ergibt sich somit wiederum, dass Σ eine Fläche 20en Grades ist. Auf ihr ist σ^3 vierfache, σ^5 fünffache Kurve, d eine zehnfache Gerade.

6. Betrachten wir das System Φ der Kurven ϱ^4, welche eine vorgegebene Ebene Φ berühren.

Auf der kubischen Kurve, welche Φ gemein hat mit der durch S_3 gelegten Fläche R^3, bilden die Flächen R^2 eine Involution I^4; demnach wird Φ berührt durch 6 Kurven ϱ^4, welche in S_3 ihr Bild haben. Analog gibt es $6\,\varrho^4$ des Systems, welche in S_5 abgebildet werden.

Demnach ist die Bildkurve eine $\Phi^{12} (S_3^6, S_5^6)$. Sie trifft eine $\lambda^5 (S_3^3, S_5^2)$ in 30 Punkten R. Der Ort der Φ berührenden ϱ^4 ist somit eine *Fläche* Φ^{30}, mit 6-fachen Kurven σ^3, σ^5 und 18-facher Geraden d.

Zwei Kurven Φ^{12} haben 72 Punkte R gemein; zwei Ebenen werden somit durch 72 ϱ^4 berührt.

7. Eine Ebene Φ durch die Gerade l schneidet die Fläche Δ^{12} noch in einer Kurve elften Grades; diese trifft l in den Stützpunkten der beiden ϱ^4, welche l zweimal schneiden, somit in 7 Punkten, wo Φ von Kurven ϱ^4 berührt wird. Der Ort der Berührungspunkte von Φ mit Kurven ϱ^4

ist daher eine Kurve Φ^7. Sie hat auf d einen 5-fachen Punkt (§ 4), demnach das Geschlecht 5. Weil die Bildkurve $\mathfrak{D}^{12}$ somit ebenfalls das Geschlecht 5 hat, gibt es auf ihr, ausser den beiden 6-fachen Punkten, 20 singuläre Punkte. Sei x die Anzahl ihrer Doppelpunkte, y die Anzahl ihrer Spitzen, also $x + y = 20$, demnach $72 - 2x - 3y$ ihre Klasse, so ist $60 - 2x - 3y$ die Anzahl ihrer Tangenten, welche nach S_3 zielen.

Diese Tangenten entsprechen offenbar den 3 Flächen R^2, welche Φ berühren und den 5 R^2, welche durch die Schnittpunkte von Φ und σ^5 gehen [1]). Also ist $2x + 3y = 52$ und $x = 8$, $y = 12$. Hieraus erhellt, dass eine Ebene durch 12 Kurven ϱ^4 *osculiert* und durch 8 Kurven ϱ^4 in je *zwei Punkten berührt* wird.

8. Ein Büschel (α^3) von kubischen Flächen, dessen Basis aus den windschiefen Geraden c, d und der Raumkurve σ^7 besteht, erzeugt mit einem Büschel (β^2) von quadratischen Flächen, dessen Basis aus c, d und deren Transversalen f, g besteht, eine *Kongruenz* von *rationalen* Raumkurven ϱ^4, welche c und d je dreimal treffen.

Ein β^2 trifft σ^7 ausserhalb c und d in 6 Punkten: demnach stützt eine ϱ^4 sich sechsmal auf σ^7; die Geraden f und g schneidet sie je in einem Punkte.

9. Es sei δ eine feste, durch d gelegte, Ebene; sie trifft c in einem Punkte C, σ^7, ausserhalb d, in den Punkten A_1, A_2, A_3. Der Büschel (α^3) bestimmt in δ den Büschel (α^2), mit Basispunkten A_k und C; der Büschel (β^2) erzeugt den Stralenbüschel (b) um C. Jede α^2 hat mit jedem Stral b einen Punkt E gemein; dieser Punkt soll als *Bild* der ϱ^4 betrachtet werden, welcher von den entsprechenden Flächen α^3 und β^2 erzeugt wird.

Die *singulären* Punkte A_k sind Bilder von je ∞^1 Kurven; diese liegen auf der durch A_k gelegten Fläche β_k^2.

Auch der *singuläre* Punkt C bildet ∞^1 Kurven ab; sie liegen auf der Fläche Γ^5, welche das Erzeugniss ist der Büschel (α^3) und (β^2), wenn diese derart projektiv auf einander bezogen sind, dass die ihnen entsprechenden a^2 und b sich in C berühren. Diese Fläche geht je zweimal durch c und d und enthält die drei Geraden $A_k C$.

10. Die Kurven ϱ^4, welche eine Gerade l treffen, bilden eine *Fläche* Λ und werden abgebildet auf die Punkte E einer Kurve λ. Diese geht zweimal durch jeden Punkt A und fünfmal durch C, ist daher eine λ^7. Als *rationale* Kurve besitzt sie noch zwei Doppelpunkte; diese sind die Bilder der beiden ϱ^4, welche l je *zweimal* treffen. Die Stützpunkte dieser Kurven bilden die gemeinschaftlichen Paare der Involutionen, welche (α^3) und (β^2) auf l bestimmen.

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[1]) Ich verdanke diese Bemerkung Herrn Dr. G. SCHAAKE.

Zwei Kurven $\lambda^7(3A^2, C^5)$ haben $7^2 - 3 \times 2^2 - 5^2$, also 12, Punkte E gemein. Es gibt daher 12 Kurven ϱ^4, welche *zwei beliebig gewählte Geraden* treffen.

Die Kurven ϱ^4, welche l schneiden, liegen auf einer *Fläche* A^{12}, mit Doppelkurve σ^7, fünffachen Geraden c, d, zwei Doppelkurven ϱ^4 und dreifachen Geraden f, g; letzteres erhellt hieraus, dass die ϱ^4, welche f in einem Punkte F treffen, auf der durch F gelegten α^3 liegen.

11. Jede Transversale r von c, d und σ^7 wird durch eine Kurve ϱ^3 zu einer Kongruenzkurve ergänzt. Die Geraden r bilden eine *Regelfläche* $(r)^6$, mit dreifachen Leitlinien c, d.

Die Flächen $(r)^6$ und A^{12} haben, ausserhalb c, d und σ^7, $6 \times 12 - 2 \times 3 \times 5 - 2 \times 7$, also 28, Geraden r gemein. Da von diesen nur 6 sich auf l stützen, wird l von 22 ergänzenden Kurven ϱ^3 getroffen. Demnach ist der *Ort der Kurven* ϱ^3 eine Fläche Σ^{22}.

Eine α^3 enthält 5 Geraden r; diese bestimmen je eine β^2; eine β^2 trifft, auf σ^7, 6 Geraden r. Vermittelst der Geraden r ergibt sich demnach eine Verwandtschaft (5, 6) zwischen (α^3) und (β^2); das Erzeugniss dieser (5, 6) ist eine Figur 28^{en} Grades, welche offenbar aus $(r)^6$ und Σ^{22} besteht.

Von den 6 Figuren (r, ϱ^3), welche auf der durch A_1 gelegten β_1^2 liegen, werden 5 in A_1 abgebildet; die sechste enthält die Geraden $A_1 C$ und die ihr entsprechende ϱ^3 wird in einen Punkt von $A_1 C$ abgebildet.

Die Kurven ϱ^3 werden somit auf die Punkte einer Kurve $\sigma^{11}(3A^5, C^5)$ abgebildet.

Diese hat mit $\lambda^7(3A^2, C^5)$ $7 \times 11 - 3 \times 2 \times 5 - 5^2$, also 22, Punkte E gemein; hieraus ergibt sich wiederum der Grad 22 der *Fläche* Σ. Ihr Schnitt mit δ setzt sich zusammen aus der Kurve σ^{11} und der 11-fachen Geraden d. Auch c ist 11-fach auf Σ^{22}, und σ^7 ist eine 5-fache Kurve.

Die Flächen Σ^{22} und A^{12} haben, ausser den vielfachen Linien c, d und σ^7, 28 Kurven ϱ^3 gemein; von diesen treffen 22 die Gerade l, indes die übrigen 6 den sich auf l stützenden Geraden r entsprechen.

12. Dem Büschel (β^2) gehören zwei Ebenenpaare an. Jedes dieser Paare wird von jeder α^3 in zwei Kegelschnitten getroffen, welche einen auf der Kante des Ebenenpaares liegenden Punkt gemein haben und eine zusammengesetzte Kongruenzkurve bilden.

Diese Systeme werden abgebildet auf die Punktreihen der Geraden b, welche nach den Punkten (df) und (dg) zielen. Jede dieser beiden Geraden enthält zwei Punkte E einer $\lambda^7(3A^2, C^5)$; demnach liegen auf einer Fläche Λ *vier* Figuren $(\varrho_1^2, \varrho_2^2)$.

13. Betrachten wir das System Ψ der ϱ^4, welche eine Ebene ψ berühren. Auf dem Kegelschnitt ψ^2, den ψ mit der Fläche β_1^2 gemein hat, bilden die durch A_1 gelegten ϱ^4 eine I^4; demnach ist A_1 ein sechsfacher Punkt der Bildkurve von Ψ. Die durch $A_2 A_3$ gelegte Fläche α^3 trifft ψ

in einer ψ^3; die auf dieser erzeugte I^4 hat 8 Doppelpunkte; A_2A_3 trägt somit 8 Bildpunkte E und die 6-fachen Bildpunkte A_2, A_3. Das Bild von Ψ ist daher eine Kurve $\psi^{20}(3A^6, C^{14})$.

Weil $\psi^{20}(3A^6, C^{14})$ und $\lambda^7(3A^2, C^5)$ offenbar $20\times7-3\times6\times2-14\times5$, also 34, Punkte E gemein haben, bilden die ϱ^4, welche ψ berühren, eine *Fläche* Ψ^{34}, mit 6-facher Kurve σ^7, 14-fachen Geraden c, d und 8-fachen Geraden f, g. (Die durch einen Punkt von f gelegte α^3 enthält in ψ eine I^4, mit 8 Doppelpunkten).

Weil zwei Kurven ψ^{20} sich in 96 Punkten E treffen, gibt es 96 Kurven ϱ^4, welche *zwei Ebenen berühren.*

14. Eine durch l gelegte Ebene ψ trifft Λ^{12} noch in einer Kurve λ^{11}; diese hat mit l die beiden Punktpaare gemein, in welchen die beiden l zweimal schneidenden ϱ^4 sich auf l stützen; demnach liegen auf l 7 Punkte in denen ψ von Kurven ϱ^4 berührt wird. Die Fläche Ψ wird somit von ψ in einer Kurve ψ^7 berührt.

Werden die in ψ belegenen Büschel (α^3) und (β^2) derart projektiv auf einander bezogen, dass entsprechende Kurven sich auf c berühren, so erzeugen sie eine Kurve 5en Grades mit 3-fachem Punkte. Somit hat ψ^7 zwei dreifache Punkte, ist daher vom Geschlecht 9. Die Bildkurve $\psi^{20}(3A^6, C^{14})$ ist ebenfalls vom Geschlecht 9, hat demnach noch $\frac{1}{2}\times19\times18-3\times\frac{1}{2}\times6\times5-\frac{1}{2}\times14\times13-9$, also 26, singuläre Punkte. Diesen entsprechen Kurven ϱ^4, welche ψ osculieren oder zweimal berühren.

Sei x die Anzahl der Doppelpunkte der ψ^{20}, demnach $26-x$ die Anzahl ihrer Spitzen; die Klasse dieser Kurve beträgt dann $20\times19-$ $-14\times13-3\times6\times5-2x-3(26-x)$, also $30+x$. Nach C zielen somit $x+2$ Tangenten.

Eine Gerade b, durch C, bestimmt auf ψ eine Kurve β^2, welche eine I^4 enthält. Wenn β^2 die Kurve σ^7 trifft, artet I^4 aus in eine I^3 und einen festen Punkt; von den 6 auf b liegenden Punkten E, sind dann zwei zusammen gefallen, wonach b Tangente der ψ^{20} ist. Wenn β^2 ein Geradenpaar ist, artet I^4 wiederum aus. Somit ist $x+2=7+3$, also $x=8$.

Die Ebene ψ wird daher von 8 Kurven ϱ^4 je *zweimal berührt* und von 18 ϱ^4 *osculiert.*

Mathematics. — *The Introduction of Coordinates into Projective Geometry.* By O. BOTTEMA. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of September 29, 1928).

The construction of Projective Geometry from a system of axioms is generally concluded by the introduction of coordinates, after which it appears that the defined geometry is identical with an analytical geometry, a geometry of coordinates.

Since the investigations of VON STAUDT different methods for this have been indicated. They come to this that the points, for the moment those of a straight line, are brought into a one-one correspondence with the numbers of a system of numbers in which operations exist that satisfy special conditions.

The properties of this system of numbers depend on the phase to which we suppose projective geometry to have been developed.

If we have introduced a complete system of axioms, such as are necessary for the construction of the "ordinary" projective geometry, the system of numbers will have all the properties of the system of the real numbers or of that of the real and the complex numbers.

For the geometry in the wider sense which we get through only postulating axioms of *Verknüpfung* but which for the rest is completely developed in the sense that the *fundamental theorem* holds good — the projective correspondence of two lines to each other is defined by three conjugated pairs of points — the corresponding system of numbers will lack all properties of order and continuity the same as the defined geometry. The operations addition and multiplication will satisfy the usual requirements. The system of numbers need not be identical with that of the real numbers but it can coincide e.g. with the set of the rational or the algebraic numbers, or it can consist of a finite number of numbers.

The modern works on projective geometry [1]) often follow this method and define the notion of coordinates before axioms of order or continuity have been introduced. In this paper too this point of view is taken.

The ways in which coordinates are introduced, are different. VON STAUDT develops an algebra of *Würfe*; SCHUR calculates with *prospectivities*, VEBLEN and YOUNG give a more direct algebra of points.

In what follows a method of introducing coordinates is sketched that rests on operations which are defined for the points of a *conic*. Accordingly

[1]) See e.g. O. VEBLEN and J. W. YOUNG, *Projective Geometry*, Volume I (1910), p. 141 sqq.

it supposes the definition and the most important properties of the conics to be given — in so far as they follow from the assumed axioms, hence with the exclusion of those of order or continuity. For the proofs of these properties we can best refer to VEBLEN and YOUNG [1]) as there only use is made of the axioms of "*alignment*" and "*extension*" and of the fundamental theorem. Of the theorems that are necessary for what follows, we mention especially the theorem of PASCAL and the theorems on projectivities on conics.

The method indicated here gives an application of the former theorem and has, perhaps, the advantage of a certain graphicalness. Before developing it we shall first point out a disadvantage. It is entirely dependent on the fundamental theorem as the theory of the conics rests on this.

By other methods we also arrive at the notion of coordinates when we omit the fundamental theorem (or the validity of the theorem of PAPPUS, which amounts to the same). In this case the system of numbers only lacks the commutativity of multiplication. (Non-Pascal geometry.) Such an extension, an introduction of coordinates in a still earlier phase of the axiomatic development, is therefore excluded here.

We choose a conic K. For the points of K we shall define a few operations. We choose two different points on K that we call resp. ∞ and o and at ∞ we draw the tangent S to K. By the *sum* $a + b$ of two points a and b of K we understand the point K that we get by the following construction. Join a b, cut the join by S and join the point of intersection to O. We call the second point of intersection of K and this join $a + b$ (Fig. 1).

If by the join of a point of K and itself we understand the tangent, the operation is *possible and one-valued* for all pairs of points of K except for the pair that consists of two points coinciding in ∞.

The addition is *commutative*: $a + b = b + a$.

This is at once evident from the construction.

The addition is *associative*: $(a + b) + c = a + (b + c)$.

Proof. Cf. Fig. 2, where a, b, c, $(a + b)$ and $(b + c)$ are indicated. We must prove that the joins of $(a + b)$ and c and of a and $(b + c)$ cut each other on S.

With a view to this consider the hexagon $a — b — c — (a + b) — O — (b + c)$. The side a, b cuts the side $(a + b)$, O on S; in the same way b, c and O, $(b + c)$ cut each other on S. According to the theorem of PASCAL also the other two opposite sides of the hexagon cut each other on S.

The addition has further the properties

$$a + 0 = 0 + a = a$$
$$a + \infty = \infty + a = \infty \quad (a \neq \infty).$$

The operation is unambiguously reversible; there is always one point x for which $a + x = b$, provided a and b do not coincide in ∞. We indicate

[1]) VEBLEN and YOUNG, l.c. p. 109 sqq.

this element by $x = b - a$ and in this way we have defined „subtraction". As a special case to any point there corresponds an opposite.

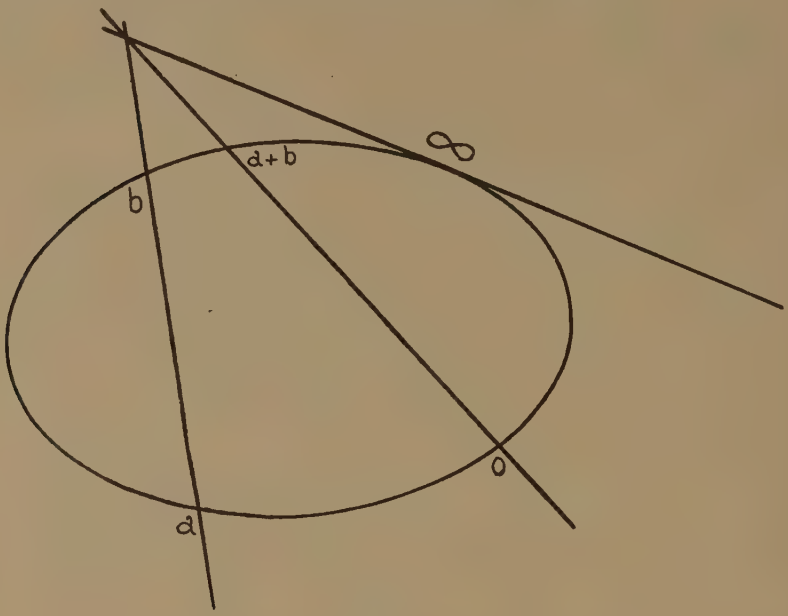


Fig. 1.

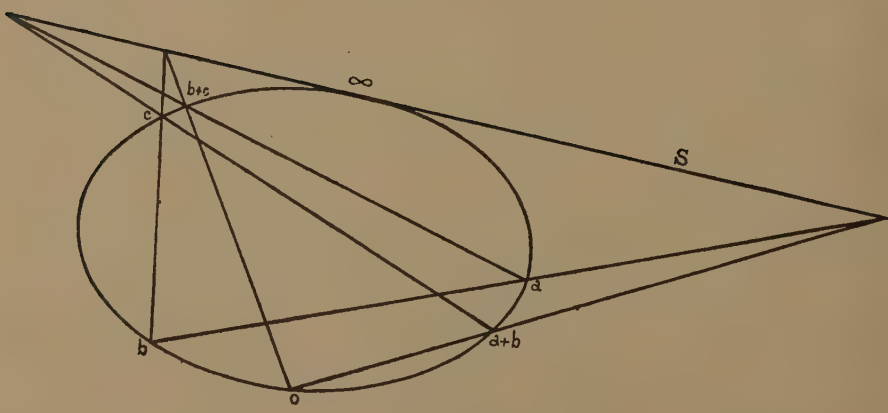


Fig. 2.

Joins of opposite points all pass through the point of intersection (A) of the tangents at ∞ and O (Fig. 3).

We shall now give a definition of the *product* of two points of K. With

a view to this we choose another point 1 on K different from ∞ and O and we join O and ∞ (p).

The construction is about the same as that for addition. Join $a\ b$, cut this line and p, join the point of intersection to 1 ; the second point of intersection of this line and K is the product ab (Fig. 4).

The multiplication is *possible* and *one-valued* for all pairs of points with the exception of the pair O, ∞.

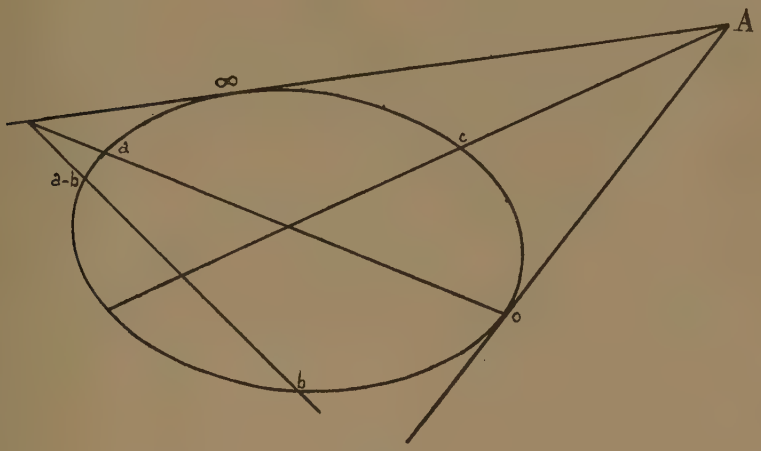


Fig. 3.

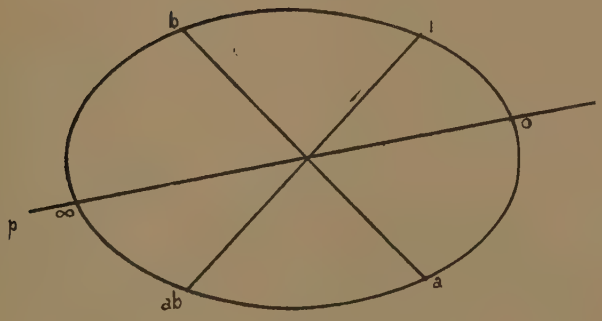


Fig. 4.

The multiplication is *commutative* as appears again directly from the construction : $ab = ba$.

It is also *associative* : $(a\ b)\ c = a\ (b\ c)$. The proof is given just as for the addition by the aid of the theorem of PASCAL. The hexagon to be considered is here

$$a — b — c — a\ b — 1 — b\ c.$$

and p is the line of PASCAL.

The multiplication has besides the following properties :

$$a \cdot \infty = \infty \cdot a = \infty \quad (a \neq 0)$$
$$a \cdot 0 = 0 \cdot a = 0 \quad (a \neq \infty)$$
$$a \cdot 1 = 1 \cdot a = a.$$

The multiplication is unambiguously reversible. If $a \neq o$ and $a \neq \infty$, there is always one point x so that.

$$a\,x = b,$$

for which we write $x = \frac{b}{a}$.

In this way the operation "division" is defined. The construction for the division is indicated in Fig. 5. In order to divide b by a we cut the line b, 1 by p and join the point of intersection to a.

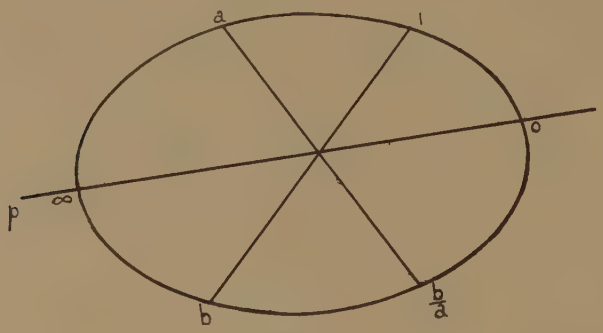


Fig. 5.

This construction can also be performed when a coincides with o or ∞. If the operation of division is extended in this way we have evidently : $\frac{a}{o} = \infty\,(a \neq o)$ and $\frac{a}{\infty} = o\,(a \neq \infty)$. The expressions $\frac{o}{o}$ and $\frac{\infty}{\infty}$ remain indefinite. To any point there corresponds one inverse point. Joins of inverse points all pass through the point of intersection (B) of p and the tangent at 1. Further the point — 1, the opposite of 1, is important. By the aid of the theory of poles of the conics it appears that the tangent at — 1 passes through B and that, accordingly, — 1 is its own inverse. Besides it appears that the rays from 1 to ∞, o, 1 and — 1 lie harmonically so that the points ∞, o, 1 and — 1 form a harmonical point quadruplet on K.

It appears accordingly that the associativity of the operations of addition and multiplication defined by us are closely connected with the theorem of PASCAL for a conic. It deserves attention that for the rest this theorem does not play any special part in the construction of projective geometry. Where in axiomatics of projective geometry the theorem of PASCAL is mentioned, the special theorem of PASCAL is always meant (for a degenerate conic),

which, in order to distinguish it from the general theorem, might more appropriately be indicated by the theorem of PAPPUS. This latter is very important as has already been pointed out: it is closely connected with the commutativity of multiplication in the system of numbers of the coordinates.

When a is a fixed point ($\neq \infty$), the correspondence of the points x' to the points x defined by the equation

$$x' = x + a$$

is a projectivity on the conic. This follows immediately from the construction for the addition, from the definition of projectivity and from the theorem that the points of a conic are projected out of two of them in projective pencils of rays. The points x are projected out of a on s and back again out of o on K.

This projectivity has two double points coinciding in ∞. Inversely it follows from the fundamental theorem that a projectivity which has two double points coinciding in ∞, and where the point o is projected in the point a, is identical with the projectivity that is fixed through $x' = x + a$.

If now we suppose the two projective point ranges x and x', in which o, x, ∞^2 are resp. associated to $a, x + a, \infty^2$, both to be projected out of a point c of K on p and back again out of 1 on K, there again appear two projective point ranges where (see Fig. 6), in connection with the definition

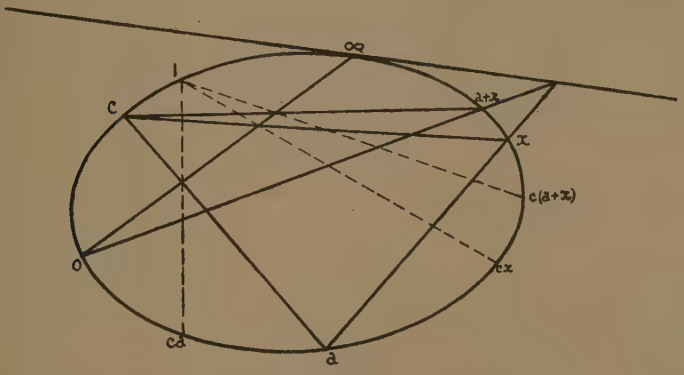


Fig. 6.

for multiplication, to o, cx, ∞^2 there correspond resp. ca, $c(x + a)$, ∞^2 so that, seen the remark just made, the equation

$$c(x + a) = cx + ca$$

holds good.

In other words the defined addition and multiplication have the *distributive* property [1]).

In this way the fundamental properties of our operations have been

[1]) I owe this simple proof to Prof. VAN DER WOUDE.

proved. These operations with points of a conic, accordingly, satisfy the rules that are valid e.g. for the real numbers. But then our aim — the introduction of coordinates — is reached.

For if l is a straight line, its points can always be projectively associated to those of K. Three arbitrary points A, B, E of l may be associated to ∞, o and 1 of K.

If in the projectivity defined in this way the point P of l corresponds to the point p of K, p is called "the coordinate of P in the system of coordinates A, B, E."

We have already seen that through $x' = x + a$ a projectivity is defined The same holds good for $x' = a\,x$ $(a \neq o)$ (projection out of a on p and back again out of 1 on K), and for $x' = \frac{1}{x}$ (involutory collineation with B as pole).

Consequently also

$$x' = \frac{ax + b}{cx + d} = \frac{a}{c} + \frac{bc - ad}{c\,(cx + d)}$$

$$(bc - ad \neq 0)$$

defines a projective correspondence between the points of l. Inversely any projectivity on l may be expressed by a broken linear function of the coordinates. For if through such a projectivity the points with coordinates a, b and c are associated to A, B, E, hence to those with coordinates ∞, o, and 1, according to the fundamental theorem it is identical with the projectivity that is defined by the equation

$$x' = \frac{(c - a)\,(x - b)}{(c - b)\,(x - a)}.$$

By the anharmonic ratio of four points (A, B, C, D) of l we shall understand the coordinate of D in the system of coordinates A, B, C. From the fundamental theorem it follows that the equality of anharmonic ratios is the necessary and sufficient condition for the projectivity of point quadruplets. It is at the same time obvious that the anharmonic ratio of four points with coordinates x_1, x_2, x_3, x_4 is equal to

$$\frac{(x_3 - x_1)\,(x_4 - x_2)}{(x_3 - x_2)\,(x_4 - x_1)}.$$

The anharmonic ratio of four harmonical points is equal to — 1 according to an above remark.

Now the further development of coordinate geometry proceeds in the same way as in the other methods. By the coordinates of a point P in a plane relative to the fundamental system A, B, C, E we understand the coordinates of the projections P_1 and P_2 (Fig. 7) in resp. the systems of coordinates $A\,C\,E^1$ and $B\,C\,E^2$, where E_1 and E_2 are the projections of the unit point.

The proof of the theorem that a straight line of the plane is represented by a linear equation and its inverse, the introduction of homogeneous

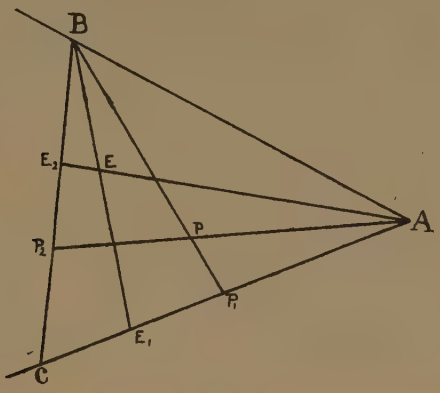


Fig. 7.

coordinates, the extension to space — it is not necessary to enter into it here.

We shall make an exception for the derivation of the equation of a conic in connection with the fact that this curve has played such a prominent part in our introduction of coordinates. Let K be the conic relative to which our operations have been defined (any other conic may be projectively associated to K). Choose the points ∞, A and o as fundamental points of the ternary system of coordinates and the point 1 of K as unit point (Fig. 8). We shall determine the coordinates of a point P of the conic and we indicate this point by t when we consider it as a number out of the system

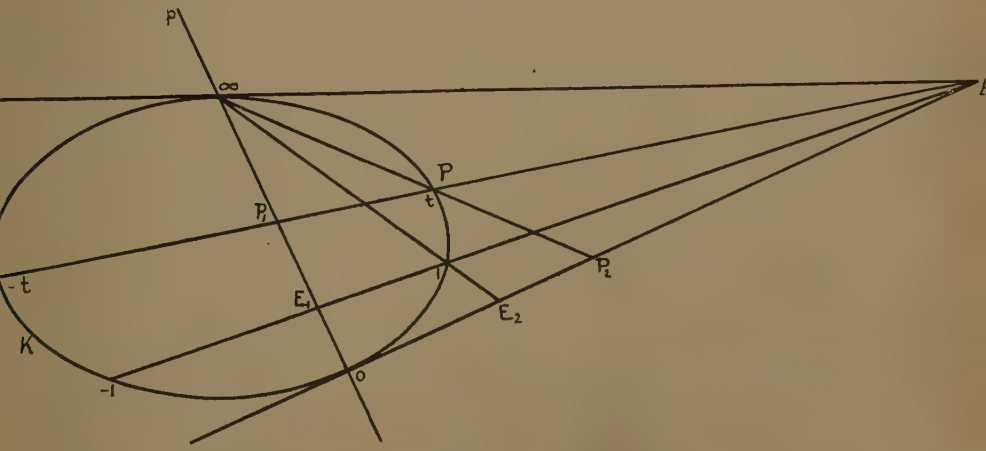


Fig. 8.

of numbers identical with the points of K. If we choose o, ∞ as X-axis, o, A as Y-axis, the x of P is accordingly the coordinate of P_1 on p in the system ∞, o, E_1. To obtain this we must associate p to K so that o and ∞ correspond to themselves and E_1 to 1. We arrive at this correspondence through projection of the points of p on K out of the point -1. P_1 is then projected in Q, which point is, accordingly, identical with the x-coordinate in question. As, however, the line AP cuts the conic besides in $-t$, we have $-1 \,.\, x = t \,.\, -t$.

$$x = t^2.$$

The Y-coordinate of P is the coordinate of P_2 relative to A, o, E_2. The Y-axis must, therefore, be projectively associated to K so that o corresponds to itself, A to ∞ and E_2 to 1. We can obtain this by projecting out of ∞ Then to P_2 P is associated, in other words $y = t$.

Hence: on a suitably chosen system of coordinates any conic can be represented by the parameter equations

$$x = t^2 \qquad y = t,$$

or by the equation $y^2 = x$. Consequently on an arbitrary system of coordinates a conic is represented by a quadratic equation.

We make a few more remarks on the operations defined on the conic K. For addition as well as for multiplication the points a and b subject to the operation had to be joined and the point of intersection of their join and a straight line (s, resp. p) had to be determined after which there followed projection out of a centre on K (o, resp. 1). For the addition the straight line was a tangent (∞), for the multiplication a chord (∞, o). Between the operations there exists this relation that the fixed chord of the latter operation joins the point of contact of the tangent and the fixed centre both used in the former operation. (Of this special position of the two figures that define the operations, the distributive property is a consequence). If we choose an arbitrary chord p, q (where $p = q$ is not excluded) and an arbitrary centre m, we can evidently relative to these data likewise define an operation between two points of K that is also commutative and associative (Fig. 9).

However it is easily seen that this can be derived from addition and multiplication (resp. the inverse operations). For if we represent the result of the operation applied to a and b by $\overline{a \mid b}$, if a is considered fixed and b variable, the correspondence

$$b' = \overline{a \mid b}$$

is projective where p and q are invariant and m is transformed into a. But this transformation is obtained by the equation

$$\overline{a \mid b} = \frac{ab(p+q) - ab\,m + pq\,m - pq(a+b)}{ab - (a+b)\,m + (p+q)\,m - pq},$$

where a, b are interchangeable with p, q, as might be expected. The special cases addition and multiplication appear when $p = q = \infty$, $m = o$,

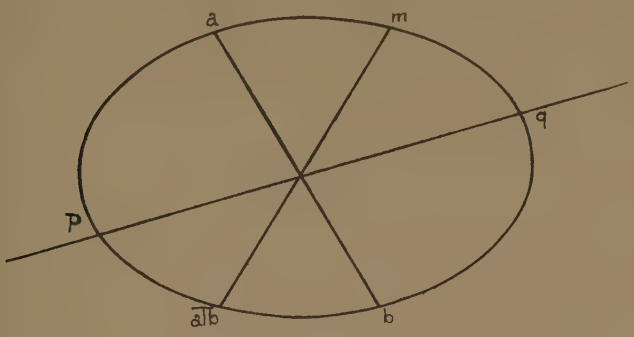


Fig. 9.

resp. $p = \infty$, $q = o$, $m = l$.

If we suppose $p = q = o$, $m = \infty$, we find the operation

$$a \mid b = \frac{ab}{a+b} \quad \text{or} \quad \frac{1}{a \mid b} = \frac{1}{a} + \frac{1}{b},$$

hence the "*harmonic addition*". It uses the tangent at o and as centre the point ∞. Consequently this operation is not only commutative and associative, but also distributive with the multiplication.

In the equation derived for $a \mid b$ we can also consider p, q, a and b as fixed and consider the equation as one that associates a new point to m by joining m to a fixed point, the intersection of p, q and a, b.

In this case the transformation gets the form

$$x' = \frac{(pq - ab)\,x + ab\,(p+q) - pq\,(a+b)}{\{(p+q) - (a+b)\}\,x + ab - pq}$$

It is obviously the general involutory projectivity between the points of the conic. The general projectivity arises by means of a curve of the second class that touches K twice.

Geology. — *Synclinal oil and unsaturated strata.* By J. VERSLUYS.

(Communicated at the meeting of December 22, 1928).

Petroleum and natural gas have, as a rule, the tendency to accumulate in the coarser strata, and in particular in the structurally highest parts of these strata. This was observed as far back as 1855 by OLDHAM, who found in a field in British India that wherever possible the oil accumulates on the anticlines. Others soon found an explanation for this phenomenon in the lower specific gravity of gas and oil as compared with that of water.

That oil and gas are mostly found in the coarser strata is to be attributed to the fact that these strata are least reduced in volume by compaction and that they form the drainage channels of the formations. The finer-grained strata, in which presumably oil and gas chiefly arise, are compressed by the burden of the overlying sediments and by orogenetic processes, the liquid (water with a little oil) and gas contained therein being forced out. Probably this is in the beginning only caused by compaction but as soon as so much gas has been generated as to be no more completely absorbed by oil and water, liquid and gas will also be expelled from the finer grained strata without further compaction. The liquid and gas then seek their way through the coarser layers, where resistance is least. Since most sediments, both coarse and fine grained, are composed of minerals which are more easily wetted by water than by oil the latter, once it has entered a coarser layer, will not readily pass out again into a finer grained layer for the area of the interface of the solid particles and water would have to decrease and the area of the interface of solid particles and oil would have to increase, which would require some work.

The same applies to gas. Thus the accumulation of oil and gas in the coarser layers — in which they have not originated — may be a result of compression and of the difference in interfacial tension with respect to the minerals of the sediments. The water pressed out of the finer grained strata by compression finds its way through the coarser layers, thereby easily entraining oil and gas. It will then leave the coarser strata again, principally in the anticlines, and find its way vertically through the finer layers, leaving the oil and gas behind because these as it has been stated heretofore, do not easily go over from a coarse to a fine grained layer, on account of the interfacial tension. In this manner the occurrence of oil and gas accumulations on anticlines can be partly accounted for without the aid of the difference in specific gravity. After anticlinal accumulation has taken place to a certain extension, however, no water can escape at the crest and further accumulation is only due to the difference in specific gravity.

Apart from the deviations from the anticlinal theory which agree with the above, there are also cases of deviations which appear to be contradictory to the generally accepted theory that the difference in specific gravity causes accumulation in the structurally highest parts. This is the not infrequent occurrence of oil in synclines. It has been supposed, however, that this could be made to agree with the theory that accumulation is a result of the difference in specific gravity, by assuming the strata with synclinal oil accumulations as being "unsaturated". By this it is meant that these strata with synclinal oil contain little or no water, so that the oil, instead of floating on the water in the structurally highest parts, precipitates in the dry strata down to the lowest parts, thereby forming accumulations in the synclines.

It seemed highly improbable to the author that at a great depth strata should occur the pores of which are not filled with a liquid or gas under high pressure, and as many data as could be obtained from literature on the subject were collected. "Unsaturated" has a very elastic meaning, but a careful study showed that many writers do, indeed, believe that it is possible for strata originally not saturated with water to be buried by younger layers and be ultimately found at a great depth still in the dry state. It has been presupposed that deposits like f.i. the sand dunes along the coast of Holland may in course of geologic time become deepseated unsaturated layers. The unsaturated state of these strata is called "connate dryness" by some authors. Thus advocates of the unsaturated strata theory assume that these layers may bear oil in the synclinal parts and air under a small (atmospheric ?) pressure in the structurally higher parts. Nevertheless in literature mention is repeatedly made of the occurrence of gas in these layers, and also of the rising of synclinal oil in wells. The supposedly unsaturated strata, therefore in addition to bearing the oil which lies on the synclines, must contain gas under pressure.

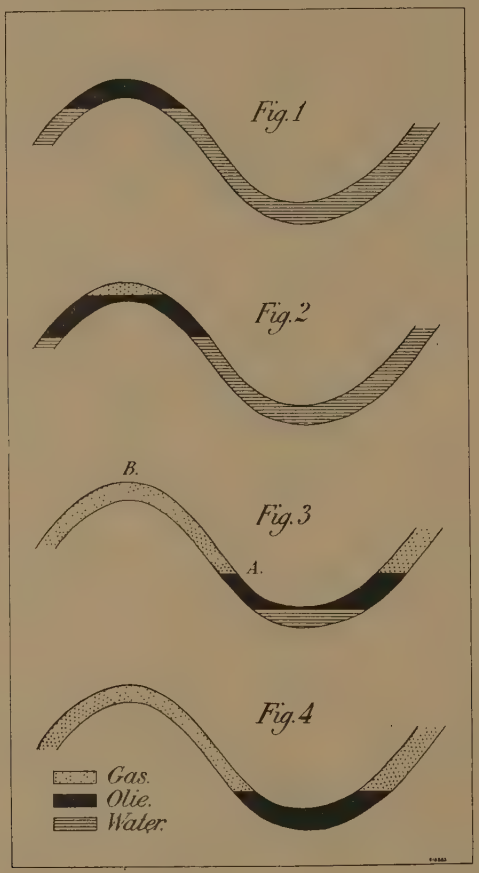
The author's attention was drawn to a description of synclinal accumulations in a treatise by Homer H. CHARLES of the Oklahoma Natural Gas Corporation, Tulsa (Okl.), The author corresponded with this geologist, who most courteously furnished him with all the particulars asked for. It appears that the structurally higher parts of the strata bearing oil in the synclines near Altoona (Kansas) contain gas under a pressure which does not differ much from the hydrostatic pressure which would prevail at the respective depth.

Whilst the anticlinal occurrence of gas and oil may be represented by figs 1 and 2, according whether free gas is present or not, the synclinal occurrence of oil will be represented in the same way by figs. 3 and 4. In synclinal oil accumulations the gas cap is greater and the water is restricted to the syncline, or is entirely absent.

Moreover it occurs that coarser layers, both on the anticline and on the syncline, contain gas exclusively. In such a case in fig. 4 the oil would have to be replaced by gas.

There is, therefore, a transition from anticlinal oil accumulations without gas caps, through those with gas caps, to the synclinal accumulations, next to which are the folded strata which bear nothing but gas.

In addition to gas, oil and water bearing layers, there are also layers



Olie = Oil

which bear only gas and water, whilst, furthermore, there are layers in which the amount of oil separating the water from the gas is of the minutest quantity. Thus in the conditions represented by figs. 1—4 the amount of oil may be very small or entirely absent. A question is whether these conditions are permanent.

The natural gas, which consists mainly of methane, is soluble in petroleum and in water, much more in the former than in the latter.

If the quantities of oil and gas do not change, a variation of the pressure

will cause a change in the volume of the gas, for if the pressure increases, more gas will dissolve in the liquids, and the volume of the not dissolved gas will be reduced by compression.

It is therefore conceivable that if more discordant sediments are deposited the pressure in the deep-seated oil and gas accumulations will increase, and the gas will be compressed. Consequently the gas caps will become smaller, so that the water in the syncline and the slopes will occupy a greater part of the layer. In the case of an oil-bearing layer coming nearer to the surface (through erosion) the gas cap might become larger and a smaller part of the layer on the syncline be occupied by water. This effect is partly counterbalanced by the changes of temperature according to the changes of depth, which also would influence the volume occupied by the gas cap. So the influence of sedimentation and erosion is probably of little importance There are, however, two other grounds for assuming that as a rule the condition in which oil and gas exist is subject to change.

If a gas cap has a considerable height the pressure of the gas will not everywhere balance that of the water in the adjacent finer-grained layers. If, for instance, there is an equilibrium at A in fig. 3, then at B there will be an excess of gaspressure and the interfacial tension will not always be able to prevent the gas from entering the finer grained layers also. Therefore it is not at all certain that gas-bearing layers or oil-bearing layers with a gas cap indeed form a condition which is geologically stationary.

Natural gas is found in very recent deposits, among other places in Holland, Suriname and the Mississippi delta, where it is to be supposed to have originated direct, and is still being developed, from the organic matter enclosed in the respective deposits. This process may also continue in older deposits. Some geologists now believe that in the course of time oil is converted, thereby giving gas. But the gas, coexisting with the oil, as a rule is principally methane and consequently by the generation of gas from the oil, the remaining oil would contain less hydrogen. So it is more probable, that gas is still formed in the finer grained strata and enters the coarser beds. In this manner the gas of the oil and gas deposits would in time increase in quantity. As remarked above, after a gas deposit or a gas cap over an oil deposit has reached a certain height, gas will be lost again in another way.

In support of the theory of the development of gas near or in already existing oil deposits is the fact that here and there the so-called reservoir pressure of the oil and gas deposits may be hundreds of pounds higher than the hydrostatic pressure which should prevail at that depth, without this pressure being accounted for by the proximity of mountains.

It is, therefore, to be assumed on the one hand that in the anticline the gas gradually disappears from the coarser layers in which it is enclosed. but that on the other hand it continues to develop so long as oil and other organic matters are present. Where there is a considerable gas cap or the oil lies in the syncline, the latter process predominates.

Apart from the gas which is found in the coarser layers, and which the American geologists term "reservoir gas", in many cases also shales contain gas. This is called "shale gas" and occurs, for instance, in black shales near the synclinal oil deposits close to Altoona, Kansas (according to a map the author received from Mr. H. H. CHARLES). Therefore there is every reason to suppose that here gas is still being developed. Either it is still formed from the organic components of the black shales, or it continues to develop from the oil which it has already pushed into the syncline. The former of these possibilities is the most acceptable.

According to the above the syclinal accumulations of petroleum show only a gradual difference from the oil deposits with gas cap, and the supposedly unsaturated strata are layers containing natural gas under a pressure governed by the same laws as those of the oil-bearing layers.

An extensive treatise dealing with the literature on the subject will be published elsewhere.

A word of thanks is due to Mr. HOMER H. CHARLES of Tulsa, to whom the author is much obliged for the information with which he was so kindly furnished.

Physics. — *Penetrating radiation.* II. By J. CLAY.

(Communicated at the meeting of December 22, 1928).

For the determination of the intensity of the penetrating radiation which was reported in these Proceedings Vol. 30, 1927, p. 1115, two electrometers, constructed according to KOLHÖRSTER by GÜNTHER and TEGETMEYER at Brunswick, were used.

When computing the results, we adopted the values of the capacity as given by the firm. At the time we lacked in Bandoeng the opportunity to measure this particular small capacity (the given values being 0 51 cm and 0.61 cm). In order to get a control on the value of this capacity, EVE's constant was determined with the help of the ionisation due to 1 m.gram *Ra* at 2 m distance. The values found were 6.4×10^9 I (ions cm^{-1} gr^{-1} sec^{-1}) and 6.6×10^9. These values were rather large, but the agreement got with the two apparatuses, made one trust the given capacity.

On a visit at Brunswick the value of the capacity was again determined; it then appeared that the method applied viz. with a condensator to HARMS, was not at all sufficiently accurate for the determination of these small capacities.

Through the kind permission of Dr. HOLST, director of the physical laboratory of the Philips' Works, I was allowed to make a capacity measurement in his laboratory with the apparatus especially built by Dr. V. D. POLL and Ir. GROENEVELD, for accurate measurement of small capacities. The method was founded on the adjusting of two circuits; the frequency of one of them was kept constant, while the frequency of the other, which held the capacity of the apparatus, was brought by means of a standard, parallel mounted condensator to such a value, that the difference in frequency of this one with the first was equal to another constant frequency. After disconnecting the capacity of the electrometer, the parallel capacity was varied, so that again the result was the same difference in frequency.

By this determination it appeared that an accuracy of 0.001 cm could be attained. As the electric system of the electrometer is mounted insulated in the middle of the ionisationchamber, the capacity is found by measuring the difference which arises when the charging connection first is brought into contact with the system and then is taken away by turning. The position which was adopted as the most accurate value, was the one in which the capacity of the charging connection did not change anymore by turning it further. In this position we may assume that the capacitive working of the system is no longer influenced by the charging-connection.

With three independent observations the values 0.287, 0.288 and 0.289 cm were obtained. An entirely different measurement was carried through at the Reichsanstalt in Berlin with the aid of Dr. ZICKNER and Dr. KOLHÖRSTER. There the capacity was measured with the help of a bridge with an alternating current of high frequency and telephone. The result of this measurement was 0.297 cm. The accuracy amounted to about 2 %. From now we adopted 0.288 as value of the capacity. Probably also other investigators, who used KOLHÖRSTER's electrometer and who had no opportunity to make accurate capacity measurements will also have found some uncertainty in the absolute value of the measured intensities. From this may be partly explained the difference of the values found.

The capacity of the electrometer *B*, which remained in Bandoeng, was after our return determined by comparing it with *A*, while both were mounted at a distance of 2 m from 1 m.gr. radium. In this way the capacity of *B* was found to be 0.315 cm, so that important corrections had to be applied to the absolute values of the intensities published in a previous paper. The values of *A* have to be multiplied with 0.57 and those of *B* with 0.50. After a series of measurements during a few weeks at Leyden, instrument *A* was filled there with argon from the Philips' Works, the latter having been kept there for about ten years and being of a sufficient purity.

It was proved before that the apparatus showed no leakage when an overpressure of 20 cm mercury was applied.

The proportion of the ionisation values with argon and with air was determined from some series of measurements and appeared to be at Leyden without armour 1.15 and with an armour of 8 cm iron 1.06. The same proportion was for a second time determined in Bandoeng in July and under the influence of *Ra*-radiation there was successively found 1.18 with armour and 1.17 without armour.

For the mean value of the ionisation proportion filled with argon and with air, we obtained in this way 1.15. Supposing that the ionisation is simply dependent on the number of electrons per unit of volume, we could expect to find for this proportion the number 1.14.

A calibration of the electrometer *A* with a quantity *Ra* of 9.17 m.gr. *Ra*-element of the physical laboratory at Leyden gave for the EVE's constant a value of 3.81×10^9 *I*, which therefore proved, that the found value of the capacity could now be trusted. After having the instrument in an armour of 8 cm iron, it appeared that only 3.8 % of the rays penetrated into the electrometer. From now the measurements were made with this armour in order to exclude the radio-activity of the surroundings.

It was stated in a former communication that it was impossible to obtain a reliable value of the residual ionisation of the electrometer in the Staszfürter saltmines. So it was of great importance to determine it in another way. To do this there was in Bandoeng an opportunity viz. to bring the

electrometer in a deserted tunnel of some waterpowerworks, where we could mount the instruments under a layer of 84 m rock. In the middle of the tunnel which was 124 m long, was a niche, in which we could place the electrometers. The electrometers were alternately armoured for an hour in iron, then left unprotected and finally an observation was made, in which the electrometer A was kept in armour for 16 hours. In the armour we found the mean value during several single hours to be 1.63 I with a deviation of 0.07 I, while for the value during those 16 hours we found 1.63 also. Out of the armour the ionisation was 4.01 I. As 3.8 % of the radioactive rays penetrates the armour, 1.52 could be considered as residual ionisation, and for the radioactive rays 2.49 I. With the instrument B, 1.69 I was found as residual ionisation with a radioactive radiation of 2.43 I.

From these results it appeared, that the value of the residual ionisation was very much lower than the previous one, which was obtained with an armour of 48 cm lead, the latter being 1.96 I.

With a view to the obtained results a new series of absorption-determinations were performed in order to determine a more accurate value for the absorptioncoefficient of lead. The electrometer was put into the iron-armour of which the absorption corresponds to that of 6.5 cm lead, (when we suppose the absorption to depend on the number of electrons per cm^3 only).

The following values were obtained

cm. lead	A	B
0	1,01	0,99
6,5	0,70	—
11,0	0,63	0,62
15,5	0,59	—
22,5	0,54	0,52
28,5	0,46	—
34,7	0,42	—

All these values were obtained from the mean of the measurements during at least two days and with an observation-time between 6 and 8 hours.

While at present for the above mentioned radiation the values 1.01 I from A and 0.99 I from B are found, the corresponding values obtained in March 1927 were respectively 1.16 I from A and 1.12 I from B. The latter values have been computed both with the corrected value of the capacity and the corrected value of the residual ionisation.

The side armouring of the instrument however was somewhat higher now than in March 1927. Therefore no conclusions can be drawn from this

difference. The only thing that is apparent is that the electrometers during this period of more than a year didn 't change appreciably.

An important question, which has to be solved first, is whether the radiation is homogenous or composite.

According to the theory of COMPTON and others on the scattering of the Röntgen- and gamma-rays, the primary rays on the earth will certainly cause a secondary radiation of less penetration and of smaller absorption-coefficient. Assuming that this secondary radiation, that is called by STEINKE [1]) „Umgebungsstrahlung" is completely absorbed by 11 cm lead, and that the primary radiation is homogeneous, we may write

$$I_1 = I_0\, e^{-\mu d_1} \quad \text{and} \quad I_2 = I_0\, e^{-\mu d_2}$$

if we take d_1 at least 11 cm.

Then $y = log \frac{I_1}{I_2} = \mu (d_2 - d_1)$.

From our observations appears, that this is approximately correct. In fig. 1, we took $d_1 = 11$ cm. Our observed values for $d_2 > d_1$ lay nearly on a straight line, for which we find $\mu = 0.17$ per cm lead. This corresponds to a wave length [2])

$$\lambda = \frac{\mu}{60} = 0{,}00028 \text{ A.E.} = 0{,}28\ X.$$

At the same time we can now determine the value of the intensity which we ought to find for the primary radiation when not absorbed by lead. When the straight line in the figure is extrapolated till $d_2 = 0$ we find 0.76 I while for $d_2 = 6.5$ cm lead we obtain 0.68 I. The observed values are 1.01 I and 0.70 I respectively. From this it appears that 0.25 I is caused by the secondary radiation or "Umgebungsstrahlung". From EINSTEIN's equation $h \frac{c}{\lambda} = eV$ and from the wavelength determined before, we may compute which are, the transformations of energy that may cause a radiation of the same wave-length. In this formula h is PLANCK's universal constant, c is the lightvelocity, e the elementary charge of the electron and finally, V the potential difference which the electron needs to acquire this energy. The question arises, which are the physical causes corresponding to those transformations. In the first place we think of the destruction of the atoms, as occurs in the radioactive processes. These quantities however are too small. In the second place there is the formation of atoms from protons and electrons, a.o. the formation of helium from hydrogen. During this process a destruction of mass occurs, the amount of which is very well

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[1]) E. STEINKE. Neue Untersuchungen über die durchdringende Hess'sche Strahlung. Z. f. Physik, Bd. **48**, p. 655 en 656.

[2]) These Proc. Vol. **30**, 1927, p. 1123.

known nowadays from ASTON's determination of the atomic-weights. When we put the mass of an atom of oxygen equal to 16, that of one atom

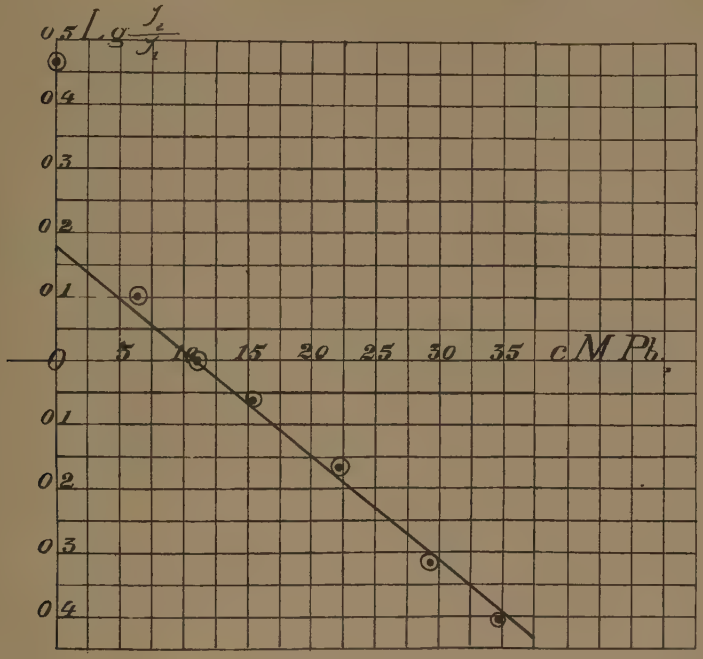

Fig. 1.

of hydrogen is 1.00778 while four atoms have a mass of 4.0311. The mass of an atom of helium is 4.00216. The real mass of an atom of hydrogen is 1.6×10^{-24} gr. Therefore the loss of mass by the formation of one helium-atom is equal to $0.0290 \cdot 1.6 \cdot 10^{-24}$ gr. According to the theory of relativity this loss of mass corresponds to a loss of energy $dE = c^2 dm$.

If this is transformed into radiation, then we have

$$dE = hV = h\frac{c}{\lambda} = c^2 dm.$$

or

$$\lambda = \frac{h}{c\,dm} = \frac{6{,}6 \cdot 10^{-27}}{3 \cdot 10^{10} \cdot 0{,}0290 \cdot 1{,}6 \cdot 10^{-24}} = 5 \cdot 10^{-12} \text{ cm.}$$

The corresponding coefficient of absorption is 0.03. This value is of the same order as the observed one. It is apparent that also other formation-processes may cause this radiation. MILLIKAN and CAMERON [1]) who from their measurement of the absorption in water conclude that the radiation

[1]) R. A. MILLIKAN and G. H. CAMERON. Scientific American. Aug. 1928, p. 136.

consists of three different components with values of the absorption-coefficients, resp. 0.028, 0.0064 and 0.0032, finally arrived at the conclusion that probably there are three different causes of radiation, which in some way are related to the formation of helium, oxygen and silicium from hydrogen.

A very remarkable fact is, that the third source of energy which is amply discussed by the astronomers, viz. the destruction of protons, corresponds to a wave-length which is considerably smaller than the one found up till now, viz. 0.014 *X* with a coefficient of absorption in lead amounting to 0.00084. Untill now no radiation of this absorption has been observed. Moreover, as MILLIKAN remarks, if this radiation really exists, its intensity cannot be large as the value of the residual ionisation only leaves a small value for the ionisation, which may be ascribed to this radiation.

The important question remains to be solved however, where this radiation comes from. KOLHÖRSTER and BÜTTNER have advocated the opinion, that the radiation should originate in the Milky way. If it is true, that it originates in a definite part of the universe, we should expect a dayly variation systematically depending on sidereal time. According to the above mentioned investigators this actually did occur. From observations made in Bandoeng in May 1927, I qualified in my previous communication this opinion as being improbable [1]). The investigations of MILLIKAN and CAMERON [2]) near lake Miguilla in Bolivia, of E. STEINKE [3]) in Davos and of HOFFMANN and LINDHOLM [4]) in Muottos Muraigl lead to the same negative conclusion. I think it more probable that the radiation originates nearer by, perhaps even in the upper layers of our atmosphere. This view is for instance confirmed by the decrease of intensity with decreasing latitude of the place of observation. During the voyage from Europe to Java on board of s.s. „Prins der Nederlanden" frequent observations were made while the instrument was 14 meter above sealevel. The observations were made without any armour and with an armour of 8 cm iron. The following consecutive values were obtained (see table following page).

From these data it is apparent that the intensity of the radiation decreases towards the equator. In the armour the gamma radiation of *Ra* was reduced to 3.8 %. On board of the steamer this radiation proves to be very small, as appears from the observations which from time to time were made without armour. By the armour, when protected at the bottom and at the side by a thick layer of lead, also the secondary penetrating radiation

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[1]) These Proc. Vol. **30**, 1927, p. 1119.

[2]) R. A. MILLIKAN and G. H. CAMERON. Nature. Jan. 1928 p. 19.

[3]) E. STEINKE. Neue Untersuchungen über die durchdringende Hess'sche Strahlung. Z. f. Physik Bd. **48**, p. 655 and 656.

[4]) G. HOFFMANN und F. LINDHOLM. Registrierbeobachtungen der Hess'schen Ultra-γ-Strahlung. Gerlands Beiträge zur Geophysik Bd. **20**, 1928, p. 12.

	I with armour	II without armour
Leiden	1,49	—
From Genua till Messina	—	1,49
Strait of Messina	1,25	—
Messina till Port Said	0,97	—
Port Said	0,99	—
Canal of Suez	0,82	1,30
Read Sea Northern part	0,92	1,25
Read Sea Southern part	0,92	—
Indian Ocean	0,86	1,04
Colombo till Sabang	0,88	—
Port of Sabang	0,87	—
Sabang till Singapore	0,85	—
Singapore till Batavia	0,82	—
Port of Batavia	0,88	—
Bandoeng (760 m above sealevel)	0,99	1,01 protected at the bottom and at the side.

is for the greater part absorbed, as is seen from the measurement in Bandoeng. On land the gammaradiation causes a larger ionisation of course, while probably the amount of secondary radiation will be larger also.

If the proportion between primary and secondary radiation may be assumed to be the same in Leyden and in Bandoeng, we find the value of the primary radiation in Leyden to be $\frac{0.76}{0.99} . 1.49\, I = 1.1\, I$. This agrees with the data given by STEINKE [1]) in Konigsberg who also gives a value of 1.1 *I*. On the other hand the value of the primary radiation in Bandoeng is 0.76 *I* with 0.25 *I* „Umgebungsstrahlung" (see before). According to MILLIKAN and CAMERON [2]) the total amount of radiation at sealevel is 1.4 *I*, while in Bandoeng it is found to be 1.01 *I*.

Bandoeng, Nov. 1928.

[1]) E. STEINKE. Z. f. Physik Bd. **48**, p. 647.
[2]) R. A. MILLIKAN and G. H. CAMERON. Nature, Jan. 1928, p. 19.